MODELING OF COMPUTER-AIDED ASSESSMENT FOR A SELECTED MATHEMATICAL TOPIC USING NEURAL NETWORKS

Hassan M. H. Mustafa

Abstract—This piece of research aims to introduce systematically, optimal investigation for one observed interdisciplinary educational phenomenon issue using Neural Networks. The adopted issue herein to be systematically investigated, optimally assessed, and realistically modeled named as educational assessment process. That issue specifically concerned with summative and formative / assessment for teaching / learning the mathematical topic called “How to solve long division problem?” In some details, introduced modeled educational assessment process associated to long division mathematical topic problem which based on sequential mathematical steps given by: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary). Presented Artificial Neural Network (ANN) model based on supervised learning paradigm (with a teacher). This model has been adopted in building up a realistic software computer package for computer aided assessment. The designed package is capable for systematic investigational analysis of two performance assessment items (summative and formative). That associated with teaching adopted mathematical topic applied to children with average age about 11 years old (at the fifth grade class level in elementary schools). Obtained results have been in agreement with average statistical variations relating learning rate values versus corresponding number of learning epochs (convergence time) at ANN modeling.

Index Terms— Computer-Aided assessment package, Long division problem, Neural Networks Modeling, Formative, and summative assessment.

I. INTRODUCTION

During the nineteenth of last century, technologies of computer, Information, and mobile devices play an essential role in how individuals work, live, play, and, more importantly, learn. Furthermore, this decade (1990-2000) is called decade of the brain as announcement in 1989 WHITE HOUSE REPORT in U.S.A. [1]. Over the past quarter century, there has been much concern about the state of science, technology, engineering, and mathematics (STEM) education in U.S. schools and its effect on the future of U.S. competitiveness in a global marketplace [2].

Recently, evolutionary computer based trend has been adopted by educationists while interacting in practical educational field with their learners. Consequently, educational organizations of all sizes — even the smallest schools and businesses — rely on recent technology trend for helping them in performing more efficient and effective operation [3]. At educational field, that approach results in the need for more efforts to ensure learning assessments focusing not only on what students learn (i.e., curriculum content) but also on how they learn (i.e., mathematical processes such as “How to solve long division problem?”) [4]. In context of learning sciences, a growing community conceiving knowledge, assessment of computer technology-mediated learning processes has been considered [5]. That in order to evaluate and improve learning performance quality specifically which concerned with assessment of a selected mathematical topics [5][6][7]. Therefore, herein a special attention paid to some research findings concerned with relation between teaching efficiently and effective application of computer technology, as follows. “Computers are transforming the way mathematicians discover, prove and communicate ideas”[8]. And “Computers and computation have changed the entire modern world, but their effects in the fields of sciences and engineering have been especially deep”[9]. Furthermore, applied mathematics has become more and more pedagogically, and computationally oriented. Accordingly, mathematical application software packages have been encouraged for using in physics, chemistry, and different branches of engineering as well as considering learning sciences’ issues [10][11][12][13]. Presented work, concerned essentially with modeling of a critical and challenging educational issue namely: summative and formative / assessment using ANN. That in order to perform both types of assessments while teaching / learning the mathematical topic “How to solve long division problem?” [6][7][14][15]. Research studies in education show that use of technology can help student learning, its use is generally affected by certain barriers. In this paper, general assessment barrier identified when integrating technology into the curriculum for instructional purposes the typically faced by K-12 schools, both in the United States.
States as well as other countries [16]. Briefly, the introduced work inspired by a strong belief that interdisciplinary combining of Artificial Neural Networks (ANN) modeling with observed challenging phenomenal educational issues for their investigational study, analysis, and evaluation. Accordingly, neural networks' theorists as well as educationalists have recently focused their attention on searching for optimal realistic modelling of some selected critical educational issues. That's results in an ANN model contributing to optimal investigation of teaching/learning performance functions such as distinct assessment procedures (formative and summative). Specifically, this piece of research considers the modeling of importance of summative assessment procedures in harmony with procedures of formative assessment by implementation of a relevant software computer assessment package [17]. Furthermore, this paper concerned with a specific mathematical topic namely: "How to solve long division problem?". By sequential processes as: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary). Therefore, herein, this research work presents a supervised ANN computer package combining two distinct educational assessment procedures (items) following after brief definitions given at [18]. Finally, after running of adopted computer assessment package, it results in some highly interesting competitive findings. For both assessment procedures (formative and summative) upon the basis of convergence (response) time as measuring learning parameter [19][20]. Conclusively, after running of realistically implemented simulation software package on computer. Interesting results have been in valuable analysis and evaluation of summative assessment procedures in harmony with procedures of formative assessment. Suggested ANN model package simulated both formative and summative assessment proceeded in sequential steps. The obtained results declares inverse proportionality of learning response time of correct learners' answers versus learning rate values. Furthermore, these results have been in agreement with average statistical variations relating learning rate values versus corresponding number of epochs (convergence time) at ANN modeling. The rest of this paper is organized as follows. At next second section, some details about two motivations of the adopted research approach are presented in two subsections (2.1, 2.2, and 2.3). In subsection 2.1, motivation concerned with ANN modeling paradigms that practically relevant to educational field (at classrooms). Additionally, the second motivation considers reforming of pedagogical approach based on computational algorithms and information technology, over the last few decades resulted in rapid improvement of teaching mathematical methodologies. The third motivation is given at sub-section (2.3), where learning assessment by its two items is briefly presented. Generalized steps to solve adopted mathematical topic adopted mathematical topic are given by a macro level flowchart, in addition to schematic diagrammatic view for a simplified interactive educational process, both are introduced at the third section. In the fourth section, mathematical formulation of the ANN model for performing long division is presented. Summarizing findings as simulation results in addition to obtained practical results for different learning rate values are given in the fifth section.

II. RESEARCH MOTIVATIONS

This research work has systematically three motivations which are briefly given at the next subsections A, B, and C as follow.

A. First Motivation

It is motivated by the announcement of Decade of the brain referring to the WHITE HOUSE REPORT in 1989 [1]. Furthermore, the overwhelming majority of neuroscientists have adopted the concept which suggests that huge number of neurons in addition to their synaptic interconnections constituting the central nervous system with its synaptic connectivity performing dominant roles for learning processes in mammals besides human [21]. More specifically, this motivation is supported by what revealed by National Institutes of Health (NIH) in USA that children in elementary school, may be qualified to learn “basic building blocks” of cognition and that after about 11 years of age, children take these building blocks and use them [22][23]. The extremely composite biological structure of human brain results in everyday behavioral learning brain functions. At the educational field, it is observable that learning process performed by the human brain is affected by the simple neuronal performance mechanism [24]. In this context, neurological researchers have recently revealed their findings about increasingly common and sophisticated role of (ANN'). Mainly, this role has been applied for systematic and realistic modeling of essential brain functions (learning and memory) [25]. Accordingly, neural network theorists as well as neurobiologists and educationalists have focused their attention on making interdisciplinary contributions to investigate observed educational phenomena associated with brain functional performance such as optimality of learning processes [5][26][27].
B. Second Motivation

This research work is motivated by what announced in USA that mathematics education has gained significant momentum as a national priority and important focus of school reform (National Mathematics Advisory Panel, 2008) [28]. Additionally, the work is originated by pedagogical approach for evaluation of mathematical education performance. By the end of this year (2012), it has been announced that a range of recording methods was documented, many of which seemed to be adaptations of mental and sensory methods of computation [29][30]. Students who used alternative methods tended to be less successful than students who used traditional algorithms. Therein, results suggest there is merit in conducting further research into the effects of using alternative written computational methods upon students' learning of mathematics. More specifically, when applying the division algorithm, students frequently made number fact errors in multiplication or subtraction [30], therein stated that: "Division methods and errors associated with alternative methods". Moreover, it is a worthy notice: presented teaching methodologies are associated with division errors which likely similar to the adopted mathematical topic therein [31]. Both were generally related to attempts to use material based models such as allocating marks in boxes in the lower grades, and guess and check multiplication or alternative splitting strategies in the higher grades. A relatively high proportion of students who did not use the standard algorithm for division relied upon diagram based methods recommended by Van de Walle et al. (2010) for double-digit by single-digit multiplication [31][32].

C. Third Motivation

In this work adopted learning assessment package considers its two basic items. Firstly, formative assessment which includes formal and informal processes teachers and students use together evidence for the purpose of improving learning. Secondly, summative assessment that provide evidence of student achievement for the purpose of making a judgment about student competence or program effectiveness. In their widely read article “Inside the Black Box,” Mr. Black and Mr. William demonstrated that improving formative assessment raises student achievement. Now they and their colleagues report on a follow-up project that has helped teachers change their practice, and students change their behavior so that everyone shares responsibility for the students’ learning[33]. Additionally, this work interestingly motivated by formative assessment and learning regulation for different mathematical conceptions published at [34]. Now assessment is used in education -with well paid attention- to quality improvement of teaching and learning, called as formative assessment or assessment for learning (A f L). Here the focus is on assessment of learning, or summative assessment, which is used to summarize what pupils know or can do at certain times in order to report achievement and progress [35].

III. GENERALIZED MODEL FOR LEARNING ASSESSMENT

A simplified flowchart for a computer teaching program is given at Figure 1. It presents teaching of long division which has been announced to be the focus of heated arguments in the world of mathematical education [12]. Some claim it is too difficult and that the children do not understand it, but rather perform it mechanically [12][13]. The introduced example for long division process has been practically applied at the fifth grade class level in elementary schools as declared in [7]. In Figure 1, a macro level flowchart describes briefly basic algorithmic steps is presented for the mathematical topic of long division process. That are: Divide, Multiply, Subtract, Bring Down, and repeat (if necessary) [12]. After classical assessment (paper and pencil) of the tested group, the obtained results of students’ educational achievement have been announced therein [7]. Subsequently herein, suggested CAA package has been used for measurement of time response as adopted parameter for both formative and summative assessment. Here focusing is considered on assessment of learning, or summative assessment, which is used to summarize what pupils know or can do at certain times in order to report achievement and progress [34]. Essentially, assessment functional model adopted in agreement with diagrammatic view for an interactive educational process, as shown at Figure 2, in the below. It illustrates generalized ANN block diagram which representing analogous simulation of both two online learning assessment paradigms. More precisely, presented model is well qualified in performing realistic simulation of either summative or formative assessment functions as follows. The vectors representing analogously inputs to the neural network as learning model at that Figure 2., are provided by environmental stimuli (unsupervised learning). The correction signal for the case of learning with a teacher is given by responses outputs of the model will be evaluated by either the environmental conditions (unsupervised learning) or by the teacher. Finally, the tutor plays a role in improving the input data (stimulating learning pattern), by reducing noise and redundancy of model pattern input by giving correction of submitted
answer. For package model presented in this work, correction of submitted answer is inherently stored according to relation between Computer Aided Learning (CAL), and -Aided Assessment (CAA) packages. [35]. Additionally, presented automated (CAA) package herein could provide both the student and the lecturer with feedback on accessed learned course material [36].

Fig. 1 A simplified macro-level flowchart describing briefly algorithmic steps for the suggested CAL package.

Fig. 2 A general schematic diagrammatic view for a simplified interactive educational process adapted from [19].
IV. MATHEMATICAL FORMULATION OF THE ANN MODEL

The presented model given at Figure 3, generally simulates two diverse learning ANN modes that analogous to supervised/unsupervised assessment. It presents realistically both paradigms: by interactive learning / teaching process, as well as other self-organized (autonomous) learning. By some details, firstly is concerned with classical (supervised by tutor) learning observed at our classrooms (face to face tutoring). Accordingly, this paradigm proceeds interactively via bidirectional communication process between tutor and his/her learner(s). However, secondly other learning paradigm performs self-organize (autonomously unsupervised) tutoring process. The mathematical formulation for the suggested ANN model is given as follows.

Fig. 3 Simplified diagram for ANN model, that realistically presents an automated (CAA) package for both (summative and formative assessment).

The error vector at any time instant (n) observed during learning processes (in case of supervised learning paradigm) is given by:

\[
\bar{e}(n) = \bar{y}(n) \cdot \bar{d}(n) \tag{1}
\]

Where

\( \bar{e}(n) \): Error correcting signal controlling adaptively

\( \bar{y}(n) \): The output signal of the model

\( \bar{d}(n) \): Numeric value(s) of the desired /objective parameter of learning process (generally as a vector).

Noting that this vector will not be taken into consideration for the case of unsupervised learning paradigm. Referring to above Figure 2, the following four equations describes dynamical learning performance.

\[
V_k(n) = X_j(n) W_{kj}^T(n) \tag{2}
\]

\[
y_k(n) = \phi(V_k(n)) = (1-e^{\lambda_k y_k(n)})/(1+e^{\lambda_k y_k(n)}) \tag{3}
\]
\[ e_k(n) = |d_k(n) - y_k(n)| \]  \hspace{1cm} (4)

\[ W_{kj}(n+1) = W_{kj}(n) + \Delta W_{kj}(n) \]  \hspace{1cm} (5)

where:  
- \( X \) ........ the input vector,  
- \( W \) ........ the weight vector,  
- \( \varphi \) ......... the activation function,  
- \( y \) ......... the output,  
- \( e_k \) .......... the error value,  
- \( \lambda \) ......... the gain factor suggested for ANN modeling,  
- \( d_k \) ........ the desired output value.

Noting that \( \Delta W_{kj}(n) \) the dynamical change of weight vector value. The above four equations (2-5), are commonly applied for both (supervised, and unsupervised) learning paradigms. For consideration to autonomously unsupervised learning paradigm: synaptic connectivity changes at any time instant \( (n) \), are given by synaptic weight vector value \( W \), which dynamically presented by equation (6) as follows:

\[ \Delta W_{kj}(n) = \eta y_k(n) x_j(n) \]  \hspace{1cm} (6)

However, for supervised learning equation (7) is considered

\[ \Delta W_{kj}(n) = \eta e_k(n) X_j(n) \]  \hspace{1cm} (7)

Where \( \eta \) is the learning rate value during unsupervised learning paradigm. This equation (6), simulates Hebbian unsupervised learning rule that relevant to simulate realistically behavioral animal learning. Noting that \( e_k(n) \) in (6) is substituted by \( y_k(n) \) at any arbitrary time instant \( (n) \) during learning process. In the assessment context, seeking and interpreting evidence for use by learners and their teachers after some consecutive time instants consecutive (\( n \)), reaches correct (desired) answer. This type of assessment called (Assessment for Learning) or equivalently Formative assessment. Unlike this type of assessment, summative assessment is known as Assessment of Learning result in an evaluation of student achievement by completely ending of learning process.

**V. RESULTS AND CONCLUSIONS**

**A. Individual Differences Outcomes**

Referring to Figure 4, the statistical distribution of obtained simulation results during summative assessment using ANN model, has been presented. This Figure illustrates various students’ outcomes considering individual differences at two distinct learning rate values: \( (\eta = 0.1 & \eta = 0.5) \). That is performed under assumption of virtual testing (by simulation) of 100 students. Simulation results seemed to have bell shapes to be in agreement with Gaussian (Natural) distribution.
A. Individual Differences Outcomes Learning Response Time

Learning Rate versus Learning Response Time Simulation results for Learning Rate versus response time parameter are given in below at Fig.5. This figure obtained after considering error correction (Formative Assessment) Learning paradigm (following equation (6) given in the above). The learning parameter associated with learning convergence time (response time) is given at Figures 6 &under assumption of 100 virtual students. Simulation results clarified bell shapes form seemed to be similarly in agreement with Gaussian (Natural) distribution. Noting that, the statistical distribution of learning convergence (response time) parameter is presumed for two different learning rate values : (\eta =0.1, and \eta =0.5).

Fig. 4 Illustrates realistic simulation results obtained after running of an ANN assessment model. These results present achievements values versus there frequency of occurrence at two suggested learning rate values (\eta =0.1& \eta =0.5).

Fig. 5 Illustrates the statistical distribution in relation with two learning rate values of parameter and learning response (convergence) time at Fig(A) & Fig(B) corresponding to two different learning rate values \eta 0.5& 0.1 respectively.
B. Learning Response Time versus Learning Rate Values ($\eta$)

Referring to Fig. 6, it is worth noting that learning rate ratio values are presented versus statistical variations (on the average) for corresponding learning convergence (response) time. That time is measured by the number of iteration cycles (epochs). Obtained output results (of response time) corresponding to the learning rate values (0.1, 0.2, 0.4, 0.6, and 0.8), are given respectively, as (330, 170, 120, 80, and 40) iteration training cycles. Conclusively, convergence time (number of training cycles) is inversely proportional to the corresponding learning rate values. Moreover, in nature creatures performing learning rate improvement by interaction with environment implies increase of their stored experience. Accordingly, experienced creatures have become capable to respond spontaneously to input environmental stimuli in optimal manner [37].

![Effect of eta increasing on learning speed](image)

Fig. 6 Illustrates simulation results presenting the average (statistical distribution) for learning response time (number of iteration cycles) for different learning rate values ($\eta$).

C. Obtained Practical Assessment Results

Referring to obtained practical results as examples shown at Figure 11 in below, the learning convergence time is 85 Sec. That time needed to reach correctly the solution for Long Division Problem Divide, Multiply, Subtract, Bring Down, and repeat (if necessary). This time observed to be the minimum learning convergence time. Accordingly, it is obvious to consider it to be in correspondence with highest learning rate value $\eta = 0.95$. Therefore, it is plotted at Figure 8 in one point as follows. Conversely, the worst learning convergence time (285 Sec.) considered to be correspondent to lowest learning rate value $\eta = 0.1$. Other different responding learning convergence time are plotted as illustrated at Figure 7.
Fig. 7 Illustrates assessment practical results after learning assessment using CAA. It presents the average (statistical distribution) for learning convergence time (number of iteration cycles) for different learning rate values ($\eta$).

Fig. 8 Basic print screen sample for an example of a mathematical Long Division problem (initial state) for formative and summative assessment.
D. Effect of Intrinsic Individual Learning Styles on Convergence Time Response

The intrinsic individual learning styles observed by the number of neurons contributing to solving correctly submitted Long Division problem. The convergence response time is varied in accordance with the individualized ability to contribute some number of neurons. The variation of learning convergence time observed to be
dependent upon neurons’ number regardless environmental learning rates. That intrinsic characteristic of individual learning styles is well illustrated at Figure 11 shown in below.

**LEARNERS’ STYLE DIFFERENCES**

![Fig 11. Illustrates simulation results for convergence time factor considering three different teaching methodologies corresponding to learning rates: 0.05, 0.1, and 0.3 (for learners’ style differences)](image)

**VI. CONCLUSIONS**

Analysis and evaluation of CAA packages for learning systems by using realistic ANN modeling is an interdisciplinary and challenging issue. It motivated by researchers integrating fields of education, cognitive science, and psychology. In the context of mathematical assessment of teaching process using CAA is rather critical and more challenging as it mainly concerned with brain functional matching ability of their preference learning styles with instructor’s mathematical teaching styles. Additionally, this work motivated by the trend suggested by M. Caudill that “if you are more biologically inspired, you will reach more optimal solution”. In other words, going towards optimal problem solving (for an engineered or scientifically based application), by implementations or simulations inspired by utilizing biologically inspired principles [38]. Conclusively, convergence time (number of training cycles) is inversely proportional to the corresponding learning rate values. Moreover, relating both figures (6&7). It results in an interesting finding concluded that: under impairment of learning environmental conditions that causes learning rate tends to have lower value [5]. Conversely, student subjected (under suggested CAA) for learning rate improvement (after successive formative assessment steps) by interaction with learning environment, implies increase of their stored experience (results in print screen at figure 10 with less learning time). Consequently, such student has become capable of responding spontaneously to input environmental stimuli in optimal manner [37]. Furthermore, some interesting findings have been announced concerning with time response analysis during performance evaluation of learning processes [39][40][41]. Above presented results of learning assessment illustrated by print screens’ approach at figures (8&9&10) provide educationalists with unbiased fair judgment tool for quantitative measurement of two learning parameters: output achievement, as well as learning convergence (time response). Moreover, the analysis and evaluation of learning improvement obtained via testing learning rate. Performance quality for any CAA module is frequently measured after investigational analysis of obtained educational field results versus learning rate values.
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AUTHOR BIOGRAPHY

HASSAN M. H. Mustafa: Born in Cairo, on October 1947. He received his B.Sc. Degree and M.Sc Degree in Electrical Engineering from Military Technical College Cairo-Egypt in 1970, and 1983 respectively. He received his Ph. D. degree at Computer Engineering and Systems in 1996 from Ain Shams University -Faculty of Engineering, Cairo –Egypt. Currently he is Associate Professor with Computer Engineering Department, Al-Baha University K.S.A. He is a member of some Scientific, Engineering, and educational technology Societies such as IIIS (International Institute of Informatics and Systemic), The Society of Digital Information and Wireless Communications (SDIWC). And at the International Association of Online Engineering IAOE. He is a senior member at International Economics Development Research Center (IEDRC) organization. Furthermore, he has been appointed as a member of technical comity for Artificial Neural Networks research work at IASTED organization during the period (2009-2012). He is an advisor at ELIXIR Journal and he has been appointed as a reviewer member at WCSIT Journal. His interest fields of research are Artificial Neural Networks, Natural Inspired Computations , and their applications for simulation , modeling and evaluation of learning processes/phenomena. He is an author / coauthor for more than 90 published publication papers & technical reports & books. All articles have been published at international specialized conferences and journals during time period from 1983 till 2014. His two E-mail addresses are: hasssan.mustafa@yahoo.com & hhasan@bu.edu.sa