Application of Robust Sliding Mode Control to Uncertain Power System Stability
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Abstract—Consider load frequency control of a power system with the communication delay and apply the sliding mode controller to this uncertain power system. The delay from the communication network is assumed to be constant time-delay type. The delay in two areas is assumed to be same. The modern power systems with industrial and commercial loads need to operate at constant frequency with reliable power. The goals of the LFC problem are to maintain constant frequency in a multi area interconnected power system. The time delay dependent stability analysis presented here, to ensure robust stability of uncertain time delay system is used to determine the upper bound of time delay for which the power system is robustly stable. Then the sliding surface is designed and the reaching mode control law is applied. The uncertain power system model with communication delay is controlled with two control strategies, namely 1) Equivalent Control Law and 2) State Feedback Control Law. Once the trajectories reach the surface, the control is switched to second control action.

Index Terms—sliding mode control, Linear Matrix Inequality (LMI), State space.

I. INTRODUCTION

Sliding mode control has many attractive features such as a fast response with asymptotic stability. The salient advantages of this method are 1) when the state is constrained to the sliding surface; sliding mode control can completely reject uncertainties that satisfy the matching condition; and 2) the high possibility of stabilizing some complex nonlinear systems which are difficult to stabilize by state feedback laws. Because of these advantages, variable structure control theory has found applications to various kinds of plants. In the present work, the problems of delay in-dependent as well as delay dependent stability analysis has been studied and subsequently robust control with sliding mode control are also investigated for uncertain time delay systems. The objective of the present work is to implement the delay dependent stability analysis of LMI approach, and sliding mode control to uncertain power system model with communication delay for load frequency control. The sliding mode control involves in applying reaching motion control law, by which the trajectories reach the surface and slide all along the surface. In sliding mode control, the control action undergoes high frequency switching leads to chattering effect. Time delays are frequently encountered in various engineering systems, such as aircraft, chemical or process control systems. Time delay often cause poor performances in control systems and may be a source of instability. Physical systems with time delay usually suffer from uncertainties which arise because of the variations in system parameters, variations in system parameters of linearized version of non linear system due to the changes of operating points, modeling errors or some ignored facts or modes. The subject of uncertain time delay systems thus received considerable amount of research interest. A linear matrix inequality (LMI) condition which is independent of time delay is derived for the existence of linear sliding surface, and by selecting suitable reaching law the reaching motion controller is designed [1]. Finally, we extend our results to the interval systems with time delay and an illustrative example problem is given for testing the design method developed.

II. SYSTEM MODEL

Consider the uncertain time-delay system of the form

\[\dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - \tau) + B(u(t) + F(\omega(t)))\]

where \(x(t) \in \mathbb{R}^n\) is the state, \(\omega(t) \in \mathbb{R}^1\) is the disturbance whose each component is bounded by the known \(\bar{\omega}_i\) i.e., \(\omega_i(t) \leq \bar{\omega}_i(t)\), \(i = 1, 2, 3, \ldots, 1\), \(u(t) \in \mathbb{R}^m\) is the control input. \(A, A_d, B\) and \(F\) are real constant matrices with appropriate dimensions and \(\text{rank}(B) = m\).

The model uncertainties are described by

\[\Delta A = \sum_{i=1}^{P} \alpha_i(t) A_i \quad |\alpha_i(t)| \leq 1\]
\[ \Delta A_d = \sum_{i=1}^{n} \beta_i(t) A_{di} \quad \left| \beta_i(t) \right| \leq 1 \]

\[ \dot{x}(t) = (A + \Delta A)x(t) + (A_d + \Delta A_d)x(t - \tau) + Bu(t) + D\Delta P_d \]  

(3)

Where \( F = (B^T B)^{-1} B^T D \); \( \omega(t) = \Delta P_d \);

\[ x(t) = [\Delta f_1, \Delta P_{m1}, \Delta P_{v1}, \Delta E_1, \Delta P_{d1}, \Delta P_{m2}, \Delta P_{v2}, \Delta E_2] \ ]; \ x(t) \] is the state vector.

\( \Delta f_1, \Delta f_2 \) = Frequency deviation in area I and II respectively

\( \Delta P_{m1}, \Delta P_{m2} \) = Generator mechanical output deviation in area I and area II respectively.

\( \Delta P_{v1}, \Delta P_{v2} \) = Governor valve position deviation in area I and area II respectively.

\( \Delta E_1, \Delta E_2 \) = Integral control deviation in area I and area II respectively.

\( \Delta P_{d1}, \Delta P_{d2} \) = Load deviation in area I and area II respectively

A. System Parameters

For two area load frequency control power system model given by (3), system parameters are as follows. All values are in p.u

Area I:

\[ T_{ch1} = 0.2s, \ T_{p1} = 0.3s, \ R_1 = 0.05, \ D_1 = 1, \ M_1 = 10, \ K_1 = 0.5; \]

Where, \( \frac{1}{T_{p1}} = \frac{B_1}{N_1}, \frac{K_{P1}}{T_{p1}} = \frac{1}{M_1} \) and \( B_1 = \frac{2}{R_1} + D_1 \)

Area II:

\[ T_{ch2} = 0.37s, \ T_{p2} = 0.4s, \ R_2 = 0.05, \ D_2 = 1.5, \ M_2 = 12, \ K_2 = 0.5; \]

Where, \( \frac{1}{T_{p2}} = \frac{B_2}{N_2}, \frac{K_{P2}}{T_{p2}} = \frac{1}{M_2} \) and \( B_2 = \frac{2}{R_2} + D_2 \)

System parameter variations are considered

\[ \frac{1}{T_{p1}} \in [0.75, 0.125], \frac{K_{P1}}{T_{p1}} \in [0.75, 0.125]. \]

\[ \frac{1}{T_{ch1}} \in [2.33, 4.23], \frac{1}{R_1} \in [19.5, 20.5]. \]

\[ \frac{1}{T_{p1}} \in [9.5, 10.5], \frac{1}{T_{p2}} \in [0.1, 0.15], \frac{K_{P1}}{T_{p1}} \in [0.075, 0.095], \frac{1}{T_{ch2}} \in [5.65, 5.95]. \]

\[ \frac{1}{R_2} \in [9.5, 20.5], \frac{1}{T_{p2}} \in [2.25, 2.5]. \]

A linear matrix inequality (LMI) condition which is dependent of time delay is derived for the existence of linear sliding surface, and by selecting suitable reaching law the reaching motion controller is designed [2]. Since the delay dependent criteria make use of the length of delays, they are less conservative than delay independent ones. Finally, we extend our results to the interval systems with time delay and an illustrative example problem is given for testing the design method developed [3].

\[ \Delta P_d = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \]
The system parameter descriptions are as follows:

\[ M_1 = \text{Moment of inertia of the generator in area I} \]
\[ M_2 = \text{Moment of inertia of the generator in area II} \]

Matrix \( A = \)

\[
\begin{bmatrix}
-\frac{1}{T_{p1}} & K_{p1} & 0 & 0 & -\frac{K_{p1}}{T_{p1}} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{T_{ch1}} & -\frac{1}{T_{ch2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{R_1T_{g1}} & 0 & -\frac{1}{T_{g1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
K_1B_1 & 0 & 0 & 0 & K_1 & 0 & 0 & 0 & 0 \\
2\pi T_1 & 0 & 0 & 0 & 0 & -2\pi T_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{K_{p2}}{T_{p2}} & -\frac{1}{T_{p1}} & \frac{K_{p2}}{T_{p2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{ch2}} & \frac{1}{T_{ch2}} & 1 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{R_2T_{g2}} & 0 & 0 & -\frac{1}{T_{g2}} & 0 \\
0 & 0 & 0 & 0 & K_2 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Matrix \( A_d = \)

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{T_{g2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Matrix \( E = \)

\[
\begin{bmatrix}
0 & 0 \\
0 & 0 \\
\frac{1}{T_{g1}} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & \frac{1}{T_{g2}} \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

Matrix \( D = \)

\[
\begin{bmatrix}
K_{r1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_{r2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\[ D_1 = \text{Area I generator damping coefficient} \]
\[ D_2 = \text{Area II generator damping coefficient} \]
\[ R_1 \text{ and } R_2 = \text{Speed droop in area I and II respectively} \]
\[ T_{ch1} \text{ and } T_{ch2} = \text{Time constant of the turbines in area I and II} \]
\[ T_1 = \text{stiffness coefficient, taken as 7.65} \]
\[ B_1 \text{ and } B_2 = \text{Frequency bias in area I and area II respectively.} \]

III. DESIGN OF CONTROLLERS

A. Sliding Surface and Reaching Motion Controller

Design of sliding mode control involves in design of sliding surface and design of reaching motion controller.

The LMI applying to LFC problem with system model (II), LMI has feasible solutions and the above power system is robustly stable up to delay \( \tau = 0.309 \) sec shown in fig. 1 and fig. 2 and designed sliding surface gain,

\[
C = \begin{bmatrix}
75.177 & -0.7567 & 0.757 & 29.0536 & 0.175 & 1.626 & 0.0735 \\
7.954 & -0.0205 & 0.6783 & 4.9432 & 0.1354 & 0.0524 & -0.0384
\end{bmatrix}
\]

\[
u(t) = -B_2^{-1}[KS + \epsilon sign(S) + [C \ I]A_dz(t) + [C \ I]A_dz(t - \tau) + diag(sign(S))(N_1 + N_2 + N_3)]
\]  \( (4) \)

Linear sliding surface \( S(t) = [C \ I]z(t) \),

From the equation (4) the reaching motion control law is designed as,

\[
u(t) = -B_2^{-1}(KS + \epsilon sign(S) + [C \ I]A_dz(t) + [C \ I]A_dz(t - \tau) + diag(sign(S))(N_1 + N_2 + N_3))
\]

Equivalent control law is developed by following formulation,

we have, \( S = [C \ I]z(t) ; \hat{S} = [C \ I]\dot{z}(t) \)

\[
\dot{S} = [C \ I](\hat{A} + \Delta \hat{A})z(t) + [C \ I]z(t - \tau) + B_2(u(t) + F\omega(t))
\]  \( (5) \)

when \( S = 0 \) implies \( \dot{S} = 0 \) i.e., when the trajectories reach the surface, by substituting \( \dot{S} = 0 \) in equation (5) we get

\[
u(t) = -B_2^{-1}[(C \ I)(\hat{A} + \Delta \hat{A})z(t) + (C \ I)(\hat{A}_d + \Delta \hat{A}_d)z(t - \tau) + B_2(u(t) + F\omega(t))]
\]

The above \( u(t) \) is called Equivalent control law and is used to maintain the trajectories on the surface. Once the trajectories reach the surface, the control is switched from reaching motion controller to equivalent control law shown in fig. 1 and fig. 2.

gain \( K_1 \) is designed and found as

\[
K_1 = \begin{bmatrix}
-3.250 & 0.097 & -0.4825 & -1.5692 & -0.163 & -0.4151 & -0.072 \\
0.212 & 0.0141 & 0.0742 & -0.93 & 0.0232 & 0.1371 & -0.0156
\end{bmatrix}
\]

and a delay bound \( \bar{\tau} = 1.7 \) sec.
IV. SIMULATION RESULTS

Fig. 1. Simulation results of states with reaching motion controller and equivalent control law (τ = 0.309)

Fig 2. Simulation result of sliding surface and control action with reaching motion controller and equivalent control law (τ = 0.309)
V. CONCLUSION

The problem of robust sliding mode control for a class of linear continuous time delay system via Lyapunov’s domain approach have been investigated. The time delay is a known constant and the parametric uncertainty considered is a modeling error type of mismatch appearing in the state. This investigation involves both the cases of delay independent as well delay dependent sufficient conditions. In both the cases a sufficient condition is derived for the existence of a linear sliding surface guaranteeing quadratic stability of the reduced order equivalent system restricted to the sliding surface, based on which the corresponding reaching motion controller is designed. Numerical examples are given to show the potential of the studied techniques. Both the reaching motion and sliding motion are found to be robust against mismatched uncertainties and matched external disturbance. A practical application of uncertain time delay system has been dealt in our present work, which includes the load frequency control of two area power system with constant time delay. It has been observed that the sliding mode control designed works satisfactorily and the dynamics of the response can be changed by tuning the reaching mode tuning parameters. In sliding mode control, the control action undergoes high frequency switching called as chattering in the sliding motion phase. To eliminate the chattering effect two different control strategies are presented namely Equivalent control law and State feedback control law.

REFERENCES


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