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Cube Difference Labeling Of some Graphs

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***Abstract:** A new labeling and a new graph called cube differencelabeling and the cube difference graph is defined. Let G be a (p, q) graph. G is said to have a cubedifference labeling if there exists a injection f: V(G) →{ 0,1, …., p-1} such that the edge set of G has assigned a weight defined by the absolute cube difference of its end-vertices, the resulting weights are distinct. A graph which admits cube difference labeling is called cube difference graph. The cube difference labeling for some graphs like paths, cycles, stars, fan graphs, wheel graphs, crowngraphs, helm graphs, dragon graphs, coconut trees and shell graphs are discussed in this paper.*

Keywords: **Cube difference labeling, cube difference graph, Crown graph, Helm graph.**

I. INTRODUCTION

A function f is a cube difference labeling of a graph G of size n if f is an injection from V(G) to the set {0,1,2, … ,n} such that , when each edge uv of G has assigned the weight $|[f(u)]^3 - [f(v)]^3|$, the resulting weights are distinct. The notion of square difference labeling was introduced by J.Shiama[4] - [6]. Graph labeling can also be applied in areas such as communication network, mobile telecommunications, and medical field.

A dynamic survey on graph labeling is regularly updated by Gallian [2] and it is published by Electronic Journal of Combinatory. The notation and terminology used in this paper are taken from [1]

Definition 1.1: Let G = (V(G),E(G)) be a graph .G is said to be cube difference labeling if there exist a injection f: V(G)→{0,1,2,….p-1} such that the induced function f^*:E(G) → N given by $f^*(u\,v) = |[f(u)]^3 - [f(v)]^3|$ is injective.

Definition 1.2:A graph which satisfies the cube difference labeling is called the cube difference graph.

Definition 1.3:A crown graph $R_n$is formed by adding to the n points $v_1,v_2, \ldots v_n$ of a cycle C_n , n more pendent points$u_1,u_2, \ldots u_n$and n more lines u_iv_i , i = 1,2, 3, . . . n for n≥ 3.

Definition 1.4:A HelmH_n ,n≥ 3 is the graph obtained from a crown $R_n$by adding a new vertex joined to every vertex of the unique cycle of the crown.

Definition 1.5: The gear graph also known as a bipartite wheel graph, is a wheel graph with a graph vertex added between each pair of adjacent graph vertices of the outer cycle. The gear graph G_n has 2n + 1 vertices and 3n edges.

II. MAIN RESULTS

Theorem 2.1: The path P_n is a cube difference graph.

Proof: Let the graph G be a path P_n.Let |V (G)| = n and |E (G)| = n-1.The mapping f: V (G) → {0, 1, …., n-1} is defined by $f(u_i) = i$, $0 \le i \le n - 1$ and the induced function f^*: E(G) → N is defined by

$f^*(u_iu_{i+1}) = 3i^2 + 3i + 1$, $0 \le i \le n - 1$ and the edge set is

$E_1 = \{ u_iu_{i+1} \;/\; 0 \le i \le n-1 \}$

Here we get all the edges with distinct weights. Hence the path P_n is a cube difference graph.

Example: 2.2: The path P_6 is a cube difference graphs.

Theorem 2.3: The cycle C_n admits a cube difference labeling.

Proof: Let the graph G be a cycleC_n.Let |V (G)| = n and |E (G)| = n. The mapping f: V (G) → {0, 1, …., n-1} is defined by $f(u_i) = i$, $0 \le i \le n - 1$ and the induced function f^*: E(G) → N is defined by

$f^*(u_iu_{i+1}) = 3i^2 + 3i + 1$, $0 \le i \le n - 1$ and $f^*(u_{n-1}\, u_0) = u_{n-1}\, u_0$

here the edge sets are

$E_1 = \{ u_iu_{i+1} \;/\; 0 \le i \le n-1 \}$ and $E_2 = \{ u_{n-1}u_0 \}$

Here we get all the edges with distinct weights. Hence the CycleC_n is a cube difference graph.

Example: 2.4: The cycle$C_6$is a cube difference graphs.

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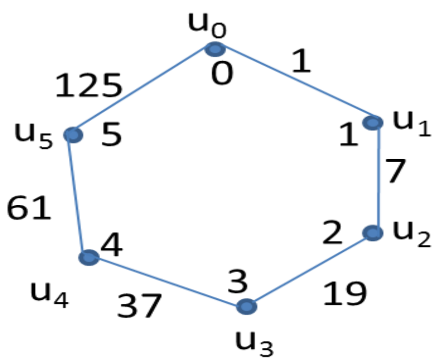


Theorem 2.5: Star graphs$K_{1,n}$ admits a cube difference labeling.

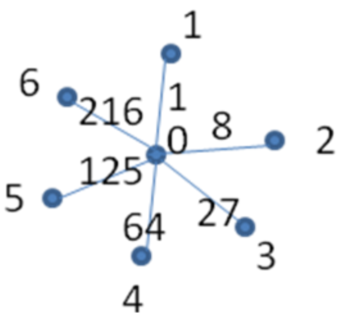
Proof:: Let the graph G be a star graph$K_{1,n}$.Let |V (G)| = n+1 and |E (G)| = n.The mapping f: V (G) → {0, 1, …., n-1} is defined by f(u) = 0 , $f(u_i) = i$, $1 \leq i \leq n$ and the induced function f^*: E(G) → N is defined by

$f^*(uu_i) = i^3$, $1 \leq i \leq n$ here the edge set is

$E_1 = \{ uu_i \ / \ 1 \leq i \leq n\}$

Here the edges are distinct. Hence the Star graphs$K_{1,n}$ admits a cube difference labeling.

Example: 2.6: TheStar graph$K_{1,6}$is a cube difference graph.



Theorem 2.7: Fan graphsF_n admits a cube difference labeling.

Proof: Let $|V (F_n)| = 2n+1$ and $|E (F_n)| = 3n$.The mapping f: V (G) → {0, 1, …., n} is defined by f(u) = 0 , $f(u_i) = i$, $1 \leq i \leq 2n$ and the induced function f^*: $E(F_n)$ → N is defined by

$f^*(uu_i) = |[f(u)]^3 - [f(u_i)]^3|$

$= i^3, 0 \leq i \leq 2n$

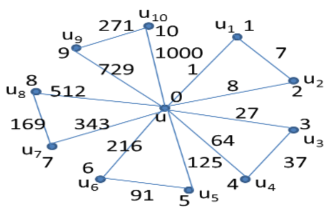
Also $f^*(u_{2i+1}u_{2i+2}) = |[f(u_{2i+1})]^3 - [f(u_{2i+2})]^3|$

$= 12 i^2 + 18i + 7, 0 \leq i \leq n$

Here the edge sets are $E_1 = \{ i^3 \ / \ 0 \leq i \leq 2n \}$and $E_2 = \{ 12 i^2 + 18i + 7 \ / \ 0 \leq i \leq n \}$

Here the edges are distinct. Hence the fan graphs F_n admits a cube difference labeling.

Example: 2.8:Fan graph F_5 is a cube difference graphs.



Theorem 2.9: Crown graphs R_n admit a cube difference labeling.

Proof: Let the graph G be a crown graph R_n Let |V (G)| =2 n and |E (G)| = 2n.The mapping f: V (G) → {0, 1, ….,2 n} is defined by $f(u_i) = i$, $1 \leq i \leq 2n-1$ and the induced function f^*: E(G) → N is defined by

$f^*(uu_i) = |[f(u)]^3 - [f(u_i)]^3|$

and here the edge sets are

$E_1 = \{ u_iu_{i+1} \ / \ 0 \leq i \leq n-1 \}$

$E_2 = \{ u_{n-1}u_0 \}$

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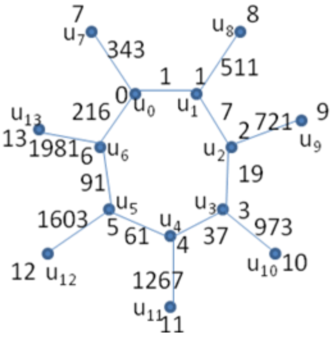
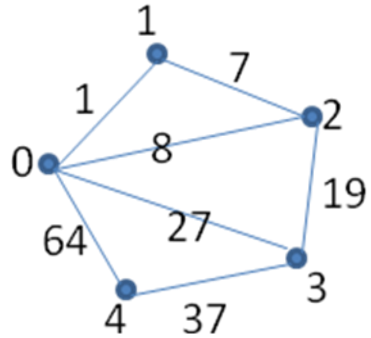
$E_3 = \{ u_i u_{n+i} / \ 0 \leq \ n+i \leq \ 2n-1 \}$
and the edge labeling are
(i)$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$
$= 3i^2 + 3i + 1, 0 \leq \ i \leq \ n-1$
(ii)$f^*(u_{n-1}u_0) = (n-1)^3$
(iii)$f^*(u_i u_{n+i}) = 21 i^2 + 147i + 343 \ , 0 \leq \ n+i \leq \ 2n-1$
Here the edges are distinct. Hence the crown graphs R_n admits a cube difference labeling
Example: 2.10: Crown graph $R_7$is a cube difference graph.



Theorem 2.11: The shell graphs $S_{n,n-3}$admit a cube difference labeling. **Proof:** Let the graph G be a Shell graph $S_{n,n-3}$.Let |V (G)| =n and |E (G)| = 2n-3.The mapping f: V (G) → {0, 1, …., n-1} is defined by $f(u_i) = i , 1 \leq \ i \leq$ n-1 and the induced function f^*: E(G) → N is defined by
$f^*(uu_i) = \ |[f(u)]^3 - [f(u_i)]^3|$
and here the edge sets are
$E_1 = \{ \ u_i u_{i+1} \ / \ 0 \leq \ i \leq \ n-2 \}$
$E_2 = \{ u_{n-1}u_0 \ \}$
$E_3 = \{ u_0 u_{i+1} \ / \ 1 \leq \ i \leq \ n-3 \}$
and the edge labeling are
(i)$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$
$= 3i^2 + 3i + 1 \ ,0 \leq \ i \leq \ n-2$
(ii)$f^*(u_{n-1}u_0) = (n-1)^3$
(iii)$f^*(u_0 u_{i+1}) = (1+i)^3, 1 \leq \ i \leq \ n-3$
Here the edges are distinct. Hence the Shell graphs $S_{n,n-3}$admits a cube difference labeling
Example: 2.12:The shell graph $S_{5,2}$ is a cube difference graph.



Theorem 2.13: The coconut tree admits a cube difference labeling.
Proof: Let $v_1,v_2, \ldots v_n$.be the vertices of a path having length i (i≥ n) and $v_{i+1},v_{i+2}, \ldots v_{i+n}$ be the pendent vertices being adjacent with v_0.
For i= 1,2,…,n the vertex labeling is defined by $f(v_j) = i , 0 \leq j \leq i$
and $f(v_h) = i + 1, i+1 \leq h \leq n$
The edge sets are
$E_1 = \{ v_j v_{j+1} \ / \ 0 \leq \ j \leq \ i \}$

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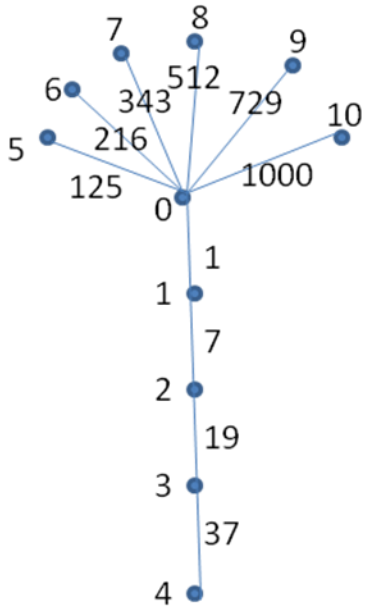
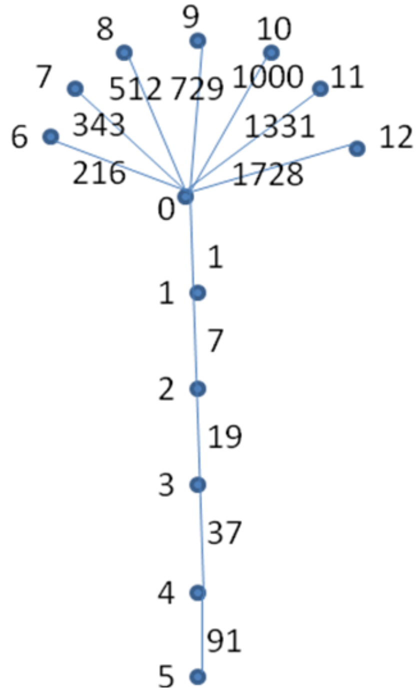
$E_2 = \{ v_h v_{h+1} \ / \ i+1 \leq h \leq n \}$
and the edge labeling are
(i)$f^*(v_j v_{j+1}) = |[f(v_j)]^3 - [f(v_{j+1})]^3|$
$=3j^2 + 3j +1$, $0 \leq j \leq i$
(ii)$f^*(v_h v_{h+1}) = (i+1)^3, i+1 \leq h \leq n$
Hence coconut tree admits cube difference labeling.
Example: 2.14:The coconut tree is a cube difference graph.





Theorem 2.15: The dragon graph $D_n(m)$admits a cube difference labeling for $n \geq 3$, $m \geq 1$
Proof: Let $u_1,u_2, . . . u_n$.be the vertices of the cycle C_n and $u_{n+1},u_2, . . . u_m$ be the edgesof the path P_m
.The mapping $f: V (G) \rightarrow \{0, 1, …., n +m -1\}$ is defined by $f(u_i) = i$, $0 \leq i \leq n +m-1$ and the induced function $f^*: E(G) \rightarrow N$ is defined by
$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$
and here the edge sets are
$E_1 = \{ u_i u_{i+1} \ / \ 0 \leq i \leq n-1 \}$
$E_2 = \{ u_{n-1} u_0 \}$
$E_3 = \{ u_i u_{n-1+i} \ / \ n-1 \leq i \leq m \}$
and the edge labeling are
(i)$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$
$= 3 i^2 + 3i +1$,$0 \leq i \leq n-1$
(ii)$f^*(u_{n-1} u_0) = (n -1)^3$
(iii)$f^*(u_i u_{n-1+i}) = 3 i^2 + 3i +1$,$n-1 \leq i \leq m$
Here the edges are distinct. Hence the dragon graphs admits a cube difference labeling
Example: 2.16:The dragon graph$D_4(3)$ is a cube difference graph.

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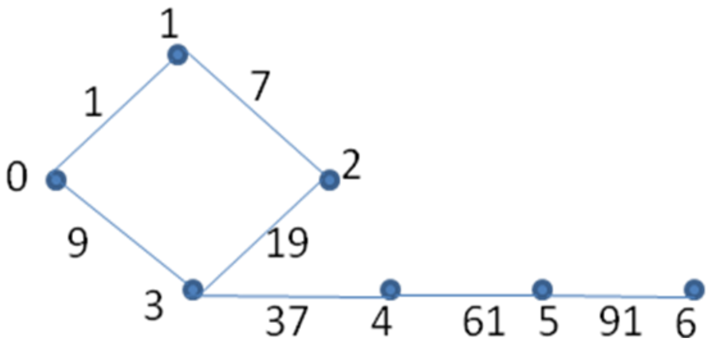


Theorem 2.17: The Helm H_n is a cube difference graph for $n \geq 3$

Proof: Let $u_0, u_1, u_2 \ldots u_{2n}$.be the vertices of the dragon graph

The mapping f: V (G) → {0, 1, ….,2n-1} is defined by $f(u_i) = i$, $0 \leq i \leq 2n-1$ and the induced function f^*: E(G) → N is defined by

$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$

and here the edge sets are

$E_1 = \{ u_0 u_i \ / \ 0 \leq i \leq n\}$

$E_2 = \{ u_i u_{i+1} \ / \ 1 \leq i \leq n \}$

$E_3 = \{ u_1 u_n \}$

$E_4 = \{ u_i u_{n+i} / \ 1 \leq i \leq 2n-1 \}$

and the edge labeling are

(i)$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$

$= 3 i^2 + 3i + 1$, $1 \leq i \leq n-1$

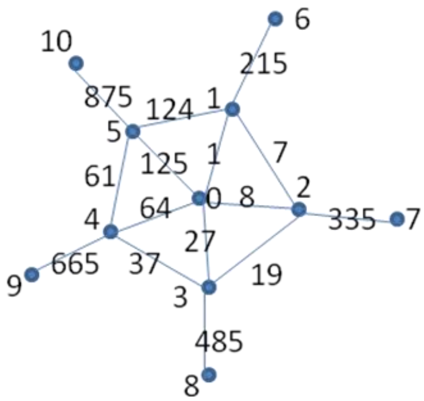
(ii)$f^*(u_0 u_i) = i^3$, $1 \leq i \leq n$

(iii)$f^*(u_i u_{n+i}) = (i+n)^3 - i^3$, $1 \leq i \leq n$

(iv)$f^*(u_1 u_n) = n^3 - 1^3$

Here the edges are distinct. Hence the Helm graphs H_n admits a cube difference labeling

Example: 2.18:The Helm H_5 is a cube difference graph.



Theorem 2.19: The Wheel W_n is a cube difference graph for n>3

Proof: Let $u_0, u_1, u_2 \ldots u_n$.be the vertices of the wheel graph

The mapping f: V (G) → {0, 1, …., n-1} is defined by $f(u_i) = i$, $0 \leq i \leq n-1$ and the induced function f^*: E(G) → N is defined by

$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$

and here the edge sets are

$E_1 = \{ u_0 u_i \ / \ 1 \leq i \leq n\}$

$E_2 = \{ u_i u_{i+1} \ / \ 1 \leq i \leq n \}$

$E_3 = \{ u_1 u_n \}$

and the edge labeling are

(i)$f^*(u_i u_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$

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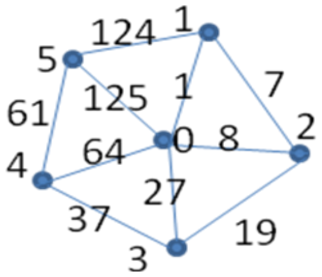
$= 3i^2 + 3i + 1 \quad ,0<i\leq n-1$

(ii)$f^*(u_0u_i) = i^3 \quad , 1\leq i\leq n$

(iii)$f^*(u_1u_n) = n^3 - 1^3$

Example: 2.20:The Wheel W_5 is a cube difference graph.



Theorem 2.21: The gear graph G_n is a cube difference graph

Proof: Let $u_0,u_1,u_2 \ldots u_n$.be the vertices of the gear graph

The mapping f: V (G) → {0, 1, …., 2n} is defined by $f(u_i) = i$, $0 \leq i\leq n-1$ and the induced function f^*: E(G) → N is defined by

$f^*(u_iu_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$

and here the edge sets are

$E_1 = \{ u_0u_i \ / \ i = 1,3,5,\ldots 2n-1\}$

$E_2 = \{ u_iu_{i+1} \ / \ 1\leq i\leq 2n-2 \}$

$E_3 = \{ u_1u_n \}$

and the edge labeling are

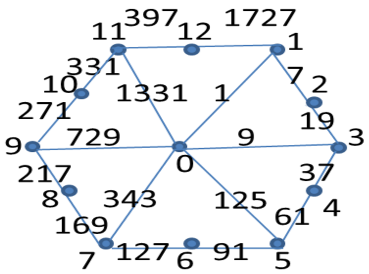
(i) $f^*(u_iu_{i+1}) = |[f(u_i)]^3 - [f(u_{i+1})]^3|$

$= 3i^2 + 3i + 1 \quad , 0<i\leq 2n-2$

(ii) $f^*(u_0u_{i+1}) = (i+1)^3$, 0, 2, 4 , … 2n

(iii) $f^*(u_1u_n) = (2n)^3 - 1^3$

Example: 2.22: The Gear graph G_6 is a cube difference graph.



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