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Evaluation of Improper Integrals in the Adaptive Integration Scheme Based On Open Type Mixed Rules

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Abstract* - An adaptive integration scheme based on open type mixed quadrature rules for evaluating improper integrals upto certain accuracy is presented. Three mixed quadrature rules are employed for framing a termination criterion of the recursive integration process. The algorithm presented in this paper is 'C' programmed. Several improper integrals are numerically evaluated by applying this scheme. The results show that this scheme is efficient and reliable for evaluation of improper integrals.

AMS subject classification: 65D30, 65D32

Keywords: **Steffensen quadrature rule, Gauss- Legendre quadrature rules, mixed quadrature rules, adaptive integration scheme.**

I. INTRODUCTION

Walter Gander and Walter Gautschi [7] have developed an adaptive integration scheme for approximation of real definite integrals. They took into account the Lobatto quadrature rule and two Kronrod extensions of Lobatto quadrature rule for framing the termination criterion. R.B.Dash and D.Das [5] have extended this idea taking into account three quadrature/mixed rules namely Clenshaw- Curtis quadrature rule and two mixed rules using Clenshaw- Curtis rule instead of using Kronrod extension to frame the termination criterion. The objective of the mixed quadrature rule [3,4,5] is construction of a symmetric quadrature rule of higher precision as a linear/convex combination of two other rules of equal lower precision.As the Steffensen 4-Point rule $\left(R_{St_4}(f)\right)$ and Gauss- Legendre 2-point rule $\left(R_{GL_2}(f)\right)$ are of precision 3, one can form a mixed rule $\left(R_{St_4GL_2}(f)\right)$ of precision 5 by taking the linear combination of these two rules. Similarly one can form a mixed quadrature rule $\left(R_{St_4GL_2GL_3}(f)\right)$ of precision 7 by taking the linear combination of the Gauss-Legendre 3-point rule $\left(R_{GL_3}(f)\right)$ and the mixed quadrature rule $\left(R_{St_4GL_2}(f)\right)$. As the mixed quadrature rule $\left(R_{St_4GL_2GL_3}(f)\right)$ and the Gauss-Legendre 4-point rule $\left(R_{GL_4}(f)\right)$ are of precision 7 one can form a mixed quadrature rule $\left(R_{St_4GL_2GL_3GL_4}(f)\right)$ of precision 9 by taking the linear combination of these two rules.

In this paper, we have used the above three open type mixed rules for framing the termination criterion. This adaptive integration scheme [1,2,6] is applied effectively for approximating some improper integrals given in the tables at the end.

(A) A simple adaptive strategy

The input to this scheme is a, b, $\in$, f, .The output is $P \approx \int_0^\infty e^{-x} f(x)\,dx$ with the error hopefully less than $\in$.

Here a=0, b=1. A simple adaptive strategy is outlined in the following four- step algorithm.

Step-1 : An approximation I_1 to $I = \int_a^b f(x)\,dx$ is computed.

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Step-2 : The interval is divided into two pieces $[a,c]$ and $[c,b]$ where $c=\frac{(a+b)}{2}$, and then $I_2 \approx \int_a^c f(x)\,dx$ and $I_3 \approx \int_c^b f(x)\,dx$ are computed.

Step 3 : $I_2 + I_3$ is compared with I_1, to estimate the error in $I_2 + I_3$.

Step 4 : If $|\text{estimated error}| \le \in/2$ (termination criterion), then $I_2 + I_3$ is accepted as a approximation to $\int_a^b f(x)\,dx$. Otherwise, the same procedure is applied to $[a,c]$ and $[c,b]$, allowing each piece a tolerance of $\in/2$. Here the approximations I_1, I_2 and I_3 are related to $R_{St_4 Gl_2}(f), R_{St_4 GL_2 GL_3}(f)$ and $R_{St_4 GL_2 GL_3 GL_4}(f)$ respectively.

II. CONSTRUCTION OF THE MIXED QUADRATURE RULE OF PRECISION FIVE

We choose the Steffensen 4-point rule $\left(R_{St_4}(f)\right)$:

$$I(f)=\int_{m-h}^{m+h} f(x)dx \approx R_{St_4}(f)=\frac{h}{12}\left[11f\left(m-\frac{3h}{5}\right)+f\left(m-\frac{h}{5}\right)+f\left(m+\frac{h}{5}\right)+11f\left(m+\frac{3h}{5}\right)\right] \quad (1)$$

and the Gauss- Legendre 2 point rule $\left(R_{GL_2}(f)\right)$:

$$I(f)=\int_{m-h}^{m+h} f(x)dx \approx R_{GL_2}(f)=h\left[f\left(m-\frac{h}{\sqrt{3}}\right)+f\left(m+\frac{h}{\sqrt{3}}\right)\right] \quad (2)$$

where $h=\frac{b-a}{2}$, $m=\frac{b+a}{2}$

Each of the rules $R_{St_4}(f)$ and $R_{GL_2}(f)$ is of precision 3. Let $E_{St_4}(f)$ and $E_{GL_2}(f)$ denote the errors in approximating the integral $I(f)$ by the rules $R_{St_4}(f)$ and $R_{GL_2}(f)$ respectively. Using Maclaurin's expansion of functions in Eqs (1) and (2), we get

$$I(f)=R_{St_4}(f)+E_{St_4}(f) \quad (3)$$

and

$$I(f)=R_{GL_2}(f)+E_{GL_2}(f) \quad (4)$$

where

$$E_{St_4}(f)=\frac{304}{375\times 5!}h^5 f^{(iv)}(m)+\frac{13136}{9375\times 7!}h^7 f^{(vi)}(m) +\frac{672992}{390625\times 9!}h^9 f^{(viii)}(m)+\frac{11004256}{5859375\times 11!}h^{11} f^{(x)}(m)+\cdots$$

$$E_{GL_2}(f)=\frac{8}{9\times 5!}h^5 f^{(iv)}(m)+\frac{40}{27\times 7!}h^7 f^{(vi)}(m) +\frac{16}{9\times 9!}h^9 f^{(viii)}(m)+\frac{464}{243\times 11!}h^{11} f^{(x)}(m)+\cdots$$

Multiplying the Eqs (3) and (4) by $\frac{1}{3}$ and $\frac{-38}{125}$ respectively and then adding the resulting equations, we obtain

$$I(f)=\frac{1}{11}\left[125R_{St_4}(f)-114R_{GL_2}(f)\right]+\frac{1}{11}\left[125E_{St_4}(f)-114E_{GL_2}(f)\right]$$

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or $$I(f)=R_{St_4GL_2}(f)+E_{St_4GL_2}(f) \tag{5}$$

where

$$R_{St_4GL_2}(f)=\frac{1}{11}\left[125R_{St_4}(f)-114R_{GL_2}(f)\right] \tag{6}$$

This is the desired mixed quadrature rule of precision 5 for the approximate evaluation of $I(f)$. The truncation error generated in this approximation is given by

$$E_{St_4GL_2}(f)=\frac{1}{11}\left[125E_{St_4}(f)-114E_{GL_2}(f)\right] \tag{7}$$

or $$E_{St_4GL_2}(f)=\frac{128}{7!\times 225}h^7 f^{(vi)}(m)+\frac{10816}{9!\times 9375}h^9 f^{(viii)}(m)+\frac{1964992}{1265625\times 11!}h^{11} f^{(x)}(m)+\cdots$$

or $$\left|E_{St_4GL_2}(f)\right|\cong\frac{128}{7!\times 225}h^7\left|f^{(vi)}(m)\right| \tag{8}$$

The rule $R_{St_4GL_2}(f)$ is called a mixed type rule of precision 5, as it is constructed from two different types of rules of the same precision .

III. CONSTRUCTION OF THE MIXED QUADRATURE RULE OF PRECISION SEVEN

We chose the mixed rule $\left(R_{St_4GL_2}(f)\right)$:

$$I(f)=\int_{m-h}^{m+h} f(x)dx\approx R_{St_4GL_2}(f)\ =\frac{1}{11}\left[125R_{St_4}(f)-114R_{GL_2}(f)\right] \tag{9}$$

and the Gauss- Legendre 3-point rule $\left(R_{GL_3}(f)\right)$:

$$I(f)=\int_{m-h}^{m+h} f(x)dx\approx R_{GL_3}(f):$$
$$=\frac{h}{9}\left[5f\left(m-\sqrt{\frac{3}{5}}h\right)+8f(m)+5f\left(m+\sqrt{\frac{3}{5}}h\right)\right] \tag{10}$$

Each of the rules $R_{St_4GL_2}(f)$ and $R_{GL_3}(f)$ is of precision 5. Let $E_{St_4GL_2}(f)$ and $E_{GL_3}(f)$ denote the errors in approximating the integral $I(f)$ by the rules $R_{St_4GL_2}(f)$ and $R_{GL_3}(f)$ respectively . Using Maclaurin's expansion of functions in Eqs (9) and (10), we get

$$I(f)=R_{St_4GL_2}(f)+E_{St_4GL_2}(f) \tag{11}$$
$$I(f)=R_{GL_3}(f)+E_{GL_3}(f) \tag{12}$$

Where

$$E_{St_4GL_2}(f)=\frac{128}{7!\times 225}h^7 f^{(vi)}(m)+\frac{10816}{9!\times 9375}h^9 f^{(viii)}(m)$$
$$+\frac{1964992}{1265625\times 11!}h^{11} f^{(x)}(m)+\cdots$$
$$E_{GL_3}(f)=\frac{8}{7!\times 25}h^7 f^{(vi)}(m)+\frac{88}{125\times 9!}h^9 f^{(viii)}(m)$$
$$+\frac{656}{625\times 11!}h^{11} f^{(x)}(m)+\cdots$$

Multiplying the equation (12) by $\frac{-16}{9}$ and then adding it with equation (11), we obtain

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$$I(f)=\frac{1}{7}\left[16R_{GL_3}(f)-9R_{St_4\,GL_2}(f)\right]+\frac{1}{7}\left[16E_{GL_3}(f)-9E_{St_4GL_2}(f)\right]$$

$$I(f)=R_{St_4\,GL_2GL_3}(f)+E_{St_4\,GL_2GL_3}(f) \tag{13}$$

where $$R_{St_4\,GL_2GL_3}(f)=\frac{1}{7}\left[16R_{GL_3}(f)-9R_{St_4\,GL_2}(f)\right] \tag{14}$$

This is the desired mixed quadrature rule of precision 7 for the approximate evaluation of $I(f)$. The truncation error generated in this approximation is given by

$$E_{St_4\,GL_2GL_3}(f)=\frac{1}{7}\left[16E_{GL_3}(f)-9E_{St_4\,GL_2}(f)\right] \tag{15}$$

or, $$E_{St_4\,GL_2GL_3}(f)=\frac{2752}{21875\times 9!}h^9 f^{(viii)}(m)+\frac{3569472}{8859375\times 11!}h^{11}f^{(x)}(m)+\cdots$$

or, $$\left|E_{St_4\,GL_2GL_3}(f)\right|\cong\frac{2752}{21875\times 9!}h^9\left|f^{(viii)}(m)\right| \tag{16}$$

The rule $R_{St_4\,GL_2GL_3}(f)$ is called a mixed type rule of precision 7, as it is constructed from two different types of rules of the same precision.

IV. CONSTRUCTION OF THE MIXED QUADRATURE RULE OF PRECISION NINE

We choose the mixed rule $\left(R_{St_4\,GL_2GL_3}(f)\right)$:

$$I(f)=\int_{m-h}^{m+h} f(x)dx \approx R_{St_4\,GL_2GL_3}(f)$$

$$=\frac{1}{7}\left[16R_{GL_3}(f)-9R_{St_4\,GL_2}(f)\right] \tag{17}$$

and the Gauss- Legendre 4- point rule $\left(R_{GL_4}(f)\right)$:

$$I(f)=\int_{m-h}^{m+h} f(x)dx \approx R_{GL_4}(f)$$

$$=\frac{h}{36}\left[\left(18+\sqrt{30}\right)\left\{f(m-\alpha h)+f(m+\alpha h)\right\}+\left(18-\sqrt{30}\right)\left\{f(m-\beta h)+f(m+\beta h)\right\}\right] \tag{18}$$

where $\alpha=\sqrt{\dfrac{3-2\sqrt{\frac{6}{5}}}{7}}$, $\beta=\sqrt{\dfrac{3+2\sqrt{\frac{6}{5}}}{7}}$

Each of the rules $R_{St_4GL_2GL_3}(f)$ and $R_{GL_4}(f)$ is of precision 7. Let $E_{St_4\,GL_2GL_3}(f)$ and $E_{GL_4}(f)$ denote the errors in approximating the integral $I(f)$ by the rules $R_{St_4\,GL_2GL_3}(f)$ and $R_{GL_4}(f)$ respectively. Using Maclaurin's expansion of functions in Eqs (17) and (18), we get

$$I(f)=R_{St_4\,GL_2GL_3}(f)+E_{St_4\,GL_2GL_3}(f) \tag{19}$$

$$I(f)=R_{GL_4}(f)+E_{GL_4}(f) \tag{20}$$

where

$$E_{St_4\,GL_2GL_3}(f)=\frac{2752}{21875\times 9!}h^9 f^{(viii)}(m)+\frac{3569472}{8859375\times 11!}h^{11}f^{(x)}(m)+\cdots$$

$$E_{GL_4}(f)=\frac{128}{1225\times 9!}h^9 f^{(viii)}(m)+\frac{2432}{8575\times 11!}h^{11}f^{(x)}(m)+\cdots$$

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Multiplying the equations (19) and (20) by $-\frac{2}{7}$ and $\frac{43}{125}$ respectively and then adding the resulting equations, we obtain

$$I(f)=\frac{1}{51}\left[301R_{GL_4}(f)-250R_{St_4GL_2GL_3}(f)\right]$$
$$+\frac{1}{51}\left[301E_{GL_4}(f)-250E_{St_4GL_2GL_2}(f)\right] \quad (21)$$

or, $I(f)=R_{St_4GL_2GL_3GL_4}(f)+E_{St_4GL_2GL_3GL_4}(f)$

where $R_{St_4GL_2GL_3GL_4}(f)=\frac{1}{51}\left[301R_{GL_4}(f)-250R_{St_4GL_2GL_3}(f)\right] \quad (22)$

This is the desired mixed quadrature rule of precision 9 for the approximate evaluation of $I(f)$. The truncation error generated in this approximation is given by

$$E_{St_4GL_2GL_3GL_4}(f)=\frac{1}{51}\left[301E_{GL_4}(f)-250E_{St_4GL_2GL_3}(f)\right] \quad (23)$$

or $E_{St_4GL_2GL_3GL_4}(f)\cong\frac{-7619325}{25302375\times 11!}h^{11}f^{(x)}(m)+\ldots$

or $\left|E_{St_4GL_2GL_3GL_4}(f)\right|\cong\frac{7619328}{25302375\times 11!}h^{11}\left|f^{(x)}(m)\right| \quad (24)$

The rule $R_{St_4GL_2GL_3GL_4}(f)$ is called a mixed type rule of precision 9, as it is constructed from two different types of rules of the same precision.

V. 'C' PROGRAMME FOR ADAPTIVE INTEGRATION SCHEME

```
#include"stdio.h"
#include"conio.h"
#include"math.h"
#include"stdlib.h"
        long double Integrand(long double E,long double A,long double B);
        long double REC_Integrand_R1(long double a,long double b);
        long double REC_Integrand_R2(long double a,long double b);
        long double REC_Integrand_R3(long double a,long double b);
        long double Integrand(long double E,long double A,long double B)
            {
                long double m,integral1,integral2,integral3,in1,in2,I,sum_register=0.0f,E1;
                static int c=1;
                printf("\n\t\t\t%d->PRESS ENTER TO PROCEED\n\n\n",c);
                c=c++;
                getch();
                m=(A+B)/2;
                integral2=REC_Integrand_R2(A,m);
                printf("\nValue of i2:%.20Lf\n",integral2);
                integral3=REC_Integrand_R3(m,B);
                printf("\nValue of i3:%.20Lf\n",integral3);
                I=integral2+integral3;
                printf("\nValue of i:%.20Lf\n",I);
                integral1=REC_Integrand_R1(A,B);
                printf("\nValue of i1:%.20Lf\n",integral1);
                E1=fabsl(I-integral1);
                printf("\nValue of E1:%.20Lf\n",E1);
                printf("\nValue of ME:%.20Lf\n",E/2);
```

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```
        if(fabsl(I-integral1) < E/2)
         {
         printf("\nError bound satisfied\n");
        return I;
         }
         else
         {
              printf("\nError  bound doesnt get satisfied\n");
              sum_register=sum_register+Integrand(E,A,m);
              return (sum_register+Integrand(E,m,B));
         }
    }
long double REC_Integrand_R2(long double a,long double b)
    {
        long double h,m,o,p,q,RSt4,RGL2,RSt4GL2;
        long double x1,x2,x3,x4,x5,x6;
        printf("\na:%.20Lf\n",a);
        printf("\nb:%.20Lf\n",b);
        h=(b-a)/2;
        m=(a+b)/2;
        o=0.6;
        p=0.2;
        q=1/sqrt(3);
        x1=1/(2*(log(1/(m-o*h)))+100);
        x2=1/(2*(log(1/(m+o*h)))+100);
        x3=1/(2*(log(1/(m-p*h)))+100);
        x4=1/(2*(log(1/(m+p*h)))+100);
        x5=1/(2*(log(1/(m-q*h)))+100);
        x6=1/(2*(log(1/(m+q*h)))+100);
        RSt4=(long double)h/12*(11*(x1+x2)+x3+x4);
        RGL2=(long double)h*(x5+x6);
        RSt4GL2=(long double)1/11*(125*RSt4-114*RGL2);
        return RSt4GL2;
    }
 long double REC_Integrand_R3(long double a,long double b)
    {
        long double h,m,o,p,q,r,RSt4,RGL2,RSt4GL2,RGL3;
        long double x1,x2,x3,x4,x5,x6,x7,x8,x9,RSt4GL2GL3;
        printf("\na:%.20Lf\n",a);
        printf("\nb:%.20Lf\n",b);
        h=(b-a)/2;
        m=(b+a)/2;
        o=0.6;
        p=0.2;
        q=1/sqrt(3);
        r=sqrt(3)/sqrt(5);
        x1=1/(2*(log(1/(m-o*h)))+100);
        x2=1/(2*(log(1/(m+o*h)))+100);
        x3=1/(2*(log(1/(m-p*h)))+100);
        x4=1/(2*(log(1/(m+p*h)))+100);
        x5=1/(2*(log(1/(m-q*h)))+100);
        x6=1/(2*(log(1/(m+q*h)))+100);
        x7=1/(2*(log(1/(m-r*h)))+100);
        x8=1/(2*(log(1/(m+r*h)))+100);
        x9=1/(2*(log(1/m))+100);
```

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```
        RSt4=(long double)h/12*(11*(x1+x2)+x3+x4);
        RGL2=(long double)h*(x5+x6);
        RSt4GL2=(long double)1/11*(125*RSt4-114*RGL2);
        RGL3=(long double)h/9*(5*(x7+x8)+8*x9);
        RSt4GL2GL3=(long double)1/7*(16*RGL3-9*RSt4GL2);
        return RSt4GL2GL3;
      }

long double REC_Integrand_R1(long double a,long double b)
    {
        long double h,m,o,p,q,r,u,v,RSt4,RGL2,RSt4GL2,RGL3,RGL4,RSt4GL2GL3GL4;
        long double  x1,x2,x3,x4,x5,x6,x7,x8,x9,x10,x11,x12,x13,RSt4GL2GL3;
        printf("\na:%.20Lf\n",a);
        printf("\nb:%.20Lf\n",b);
        h=(b-a)/2;
        m=(a+b)/2;
        o=0.6;
        p=0.2;
        q=1/sqrt(3);
        r=sqrt(3)/sqrt(5);
        u=sqrt((3-2*sqrt(1.2))/7);
        v=sqrt((3+2*sqrt(1.2))/7);
        x1=1/(2*(log(1/(m-o*h)))+100);
        x2=1/(2*(log(1/(m+o*h)))+100);
        x3=1/(2*(log(1/(m-p*h)))+100);
        x4=1/(2*(log(1/(m+p*h)))+100);
        x5=1/(2*(log(1/(m-q*h)))+100);
        x6=1/(2*(log(1/(m+q*h)))+100);
        x7=1/(2*(log(1/(m-r*h)))+100);
        x8=1/(2*(log(1/(m+r*h)))+100);
        x9=1/(2*(log(1/m))+100);
        x10=1/(2*(log(1/(m-u*h)))+100);
        x11=1/(2*(log(1/(m+u*h)))+100);
        x12=1/(2*(log(1/(m-v*h)))+100);
        x13=1/(2*(log(1/(m+v*h)))+100);
        RSt4=(long double)h/12*(11*(x1+x2)+x3+x4);
        RGL2=(long double)h*(x5+x6);
        RSt4GL2=(long double)1/11*(125*RSt4-114*RGL2);
        RGL3=(long double)h/9*(5*(x7+x8)+8*x9);
        RGL4=(long double)h/36*((18+sqrt(30))*(x10+x11)+(18-sqrt(30))*(x12+x13));
        RSt4GL2GL3=(long double)1/7*(16*RGL3-9*RSt4GL2);
        RSt4GL2GL3GL4=(long double)1/51*(301*RGL4-250*RSt4GL2GL3);
        return RSt4GL2GL3GL4;
    }
void main()
    {
        long double A,B,E,Ev,FErr;
        int ch;
        long double Ans;
        printf("\t\t\tProgram Adaptive quadrature:\n");
        clrscr();
        while(1)
        {
            clrscr();
            printf("\n\n\n\t 1.Enter the value of the nodes A,B,PrescribedtoleranceEAndExact val");
```

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```
        printf("\n\n\n\t\t 2.Integrating the given function");
        printf("\n\n\n\t\t 3.Quit");
        printf("\n\n\n\n\t\t\tEnter ur choice:");
        scanf("%d",&ch);
        clrscr();
        switch(ch)
            {
                    case 1:printf("\n\t\t Enter the value of A:");
                    scanf("%Lf",&A);
                    printf("\n\t\t Enter the value of B:");
                    scanf("%Lf",&B);
                    printf("\n\t\t Enter the value of E:");
                    scanf("%Lf",&E);
                    printf("\n\t\tEnter the Exact Value Ev:");
                    scanf("%Lf",&Ev);
                    break;
                    case 2:
                    Ans=Integrand(E,A,B);
                    printf("\n\nAns:%.15Lf\n",Ans);
                    FErr=fabsl(Ev-Ans);
                    printf("\n\nFinal Error:%.15Lf\n",FErr);
                    break;
                    case 3:
                    exit(0);
                    default :
                    printf("\nInvalid choice");
                    break;
            }
        printf("\n\t\t\t\tPRESS ENTER");
        getch();
    }
}
```

VI. NUMERICAL VERIFICATION

Table I: Approximation of some improper integrals in the whole-interval method

Integrals	Exact value	Approximate value		
		$R_{St_4}(f)$	$R_{GL_4}(f)$	$R_{St_4GL_2}(f)$
$I_1=\int_0^\infty e^{-x}x^2\,dx$	2	1.2558	1.2361	1.4600
$I_2=\int_0^\infty e^{-x}\cos x\,dx$	0.5	0.4909	0.4941	0.4576
$I_3=\int_0^\infty \frac{e^{-x}}{\sqrt{x}}dx$	$\sqrt{\pi}\approx 1.77245385$	1.4334	1.4272	1.4969
$I_4=\int_0^\infty \frac{e^{-x}}{1+x^4}dx$	0.63047783	0.5800	0.5715	0.6687
$I_5=\int_0^\infty e^{-x}\sin x\,dx$	0.5	0.6128	0.6175	0.5643

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$I_6 = \int_0^\infty e^{-x} \frac{\sin x}{x} dx$	$\frac{\pi}{4} \approx 0.7853981634$	0.8150	0.8169	0.7954
$I_7 = \int_0^\infty \frac{e^{-x}}{1+x^2} dx$	0.62144962	0.6199	0.6196	0.6225

Table II: Approximation of some improper integrals in the whole-interval method

Integrals	Exact value	Approximate value			
		$R_{GL_3}(f)$	$R_{GL_4}(f)$	$R_{St_4GL_2GL_3}(f)$	$R_{St_4GL_2GL_3GL_4}(f)$
$I_1 = \int_0^\infty e^{-x} x^2 dx$	2	1.5412	1.6914	1.6456	1.9158
$I_2 = \int_0^\infty e^{-x} \cos x\, dx$	0.5	0.4580	0.4644	0.4585	0.4929
$I_3 = \int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx$	1.77245385	1.5251	1.5797	1.5614	1.6697
$I_4 = \int_0^\infty \frac{e^{-x}}{1+x^4} dx$	0.63047783	0.6505	0.6250	0.6271	0.6150
$I_5 = \int_0^\infty e^{-x} \sin x\, dx$	0.5	0.5444	0.5109	0.5188	0.4722
$I_6 = \int_0^\infty e^{-x} \frac{\sin x}{x} dx$	0.7853981634	0.7909	0.7842	0.7851	0.7798
$I_7 = \int_0^\infty \frac{e^{-x}}{1+x^2} dx$	0.62144962	0.6222	0.6217	0.6218	0.621406

Table – III: approximation of some improper integrals in the adaptive integration scheme involving Gauss-Legendre rules $\left(R_{GL_2}(f), R_{GL_3}(f), R_{GL_4}(f)\right)$

Integrals	Exact value (I)	Approximate value (P)	No. of steps required	Error |P-I|
$I_1 = \int_0^\infty e^{-x} x^2 dx$	2	1.999996	67	0.000003
$I_2 = \int_0^\infty e^{-x} \cos x\, dx$	0.5	0.5000005	45	0.0000005
$I_3 = \int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx$	1.77245385	1.7724486	89	0.000005
$I_4 = \int_0^\infty \frac{e^{-x}}{1+x^4} dx$	0.63047783	0.6304786	15	0.0000008

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$I_5 = \int_0^\infty e^{-x} \sin x \, dx$	0.5	0.5000005	39	0.0000005
$I_6 = \int_0^\infty e^{-x} \frac{\sin x}{x} dx$	0.7853981634	0.7853968	29	0.000001
$I_7 = \int_0^\infty \frac{e^{-x}}{1+x^2} dx$	0.62144962	0.6214504	17	0.0000007

Here the prescribed tolerance $(\in) = 0.000001$

Table- IV : Approximation of some improper integrals in the adaptive integration scheme involving mixed rules

Integrals	Exact value (I)	Approximate value (P)	No. of steps required	Error |P-I|
$I_1 = \int_0^\infty e^{-x} x^2 \, dx$	2	1.999998	47	0.000001
$I_2 = \int_0^\infty e^{-x} \cos x \, dx$	0.5	0.499999908	35	0.00000009
$I_3 = \int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx$	1.77245385	1.77245303	87	0.0000008
$I_4 = \int_0^\infty \frac{e^{-x}}{1+x^4} dx$	0.63047783	0.63047806	9	0.0000002
$I_5 = \int_0^\infty e^{-x} \sin x \, dx$	0.5	0.5000010	31	0.000001
$I_6 = \int_0^\infty e^{-x} \frac{\sin x}{x} dx$	0.7853981634	0.7853965	21	0.000001
$I_7 = \int_0^\infty \frac{e^{-x}}{1+x^2} dx$	0.62144962	0.6214499	17	0.0000002

Here the prescribed tolerance $(\in) = 0.000001$

VII. CONCLUSION

In this paper we have presented an adaptive integration scheme involving open type mixed quadrature rules for approximate evaluation of improper integrals. We have taken three open type mixed rules for framing the termination criterion in the adaptive integration scheme .From tables 1 and 2, we observe that the mixed quadrature rules yield better results than its constituent rules. Tables 3 and 4 show that in the adaptive integration scheme, the mixed quadrature rules involve less number of steps in approximating improper integrals and yield better results than its constituent rules.

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