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A Proposed Statistical Procedure for Assessment of Strength of Concrete from Low Sample Size Using Monte Carlo Simulation

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Abstract— ***This paper focuses on the procedure of statistical assessment of test results in reference to the strength development of self compacting concrete and normally compacting concrete. A self compacting concrete (SCC) and a normally compacting concrete (NCC) with similar ultimate compressive strength were developed. The concrete cubes were tested at 7, 28, 60, 90, 120 and 150 days after normal water curing. For each case 10 samples were tested and the test results were recorded for each sample on as obtained basis. The problem of low sample size in reliable estimation of statistical parameters is addressed through Monte Carlo simulation technique. A new concept called "Root Mean Squared Spacing (RMSS)" is introduced to estimate adequate sample size following the principle of "Random Walk". The analysis was carried out using MATLAB software. Significance of conventional parameters like F-ratio, Signal to Noise ratio, etc in the analysis of concrete strength is discussed. The new proposed approach based on simulation technique facilitates to overcome the problem of low sample size using Monte Carlo and RMSS methods. It is shown that comparative analysis of test results can provide better meaningful understanding if systematic approach is adopted through application of conventional statistical parameters along with the new proposed procedure.***

Keywords: **Concrete, Monte Carlo method, Root mean squared spacing, Sample Size, Strength.**

LIST OF NOTATIONS

S/N – Signal to Noise ratio, RMSS - Root Mean Squared Spacing, μ - Mean Value, σ - Standard Deviation,

N – No of samples, s – Sum of Squares of test result, α - Sequence Coefficient, CDF - Cumulative Density Function, F- Value from F-Distribution, F_{Cri} - Critical F value for a given significance level.

I. INTRODUCTION

Statistical analysis of test results of concrete is significant to understand if the hardened properties of concrete can be reliably predicted by the existing formulations [1]-[3]. Several researchers have used artificial neural network to predict 28 days compressive strength of self compacting concrete [4].The model was developed from literature data and was applicable to the experimental data, with bottom ash as partial replacement of sand. Similar ANN techniques along with regression analysis were reported for prediction of compressive strength of vacuum processed concretes [5].

Statistical study on the variability of the mechanical properties of hardened self compacting concrete including compressive strength was conducted by a few investigators [6]-[7]. The variability study was done in the same range than the expected for normally compacting concrete with in a confidence level of 95%. Ramadoss and Nagamani applied statistical methods on compressive strength of high performance steel fiber reinforce concrete to develop model for quantification of the effect of fiber content on compressive strength [8]. It was reported that the estimated strength from the models differed from the actual values within + 3.2 % and -3.2 %. Equation was also proposed to estimate the effect of size of the concrete specimens.

An extensive study was carried out to conduct a statistical analysis of the compressive strength of light weight aggregate concrete of Benicia Martine Bridge in California [9]. The probability of compressive strength to fall below the minimum observed strength was increased three times at five years when compared to those observed at 35 days. The probability of compressive strength to lie above the maximum value was increased by 1.23 % at five years. Study based on statistical models was developed to understand the influence of key mixture parameters on hardened properties of SCC [10]. These responses included compressive strength at 3, 7 and 28 days and modulus of elasticity. Full quadratic models for all the cases depicted higher correlation coefficient, adjusted correlation coefficient, less level of significance and sum of square error from the linear interaction full quadratic and pure quadratic models. Experimental and software aided techniques are reported as useful means by several researchers for performance evaluation of concrete mixture in durable concrete structures [11]. Application of self adaptive genetic algorithm for optimal proportioning of concrete aggregates is reported by a few researchers [12].

Considerable body of research focuses on the distribution characteristics of compressive strength of concrete [13]. The statistical analysis indicated that the strength of most of the types of concrete followed normal distribution with very rare exception.

A few papers described a number of statistical methods to estimate the in-place compressive strength of concrete expressed in terms of standard specimens [14]. Probable existence of a linear correlation as a function of sample size was discussed. A criterion was proposed for the choice of the number of points to be used in the regression analysis. A method was also proposed to estimate the equivalent number of replicated test performed on standard specimen. Williams et al evaluated the feasibility of using sampling methods in selecting test locations, managing data collected and describing test results [15]. The results could effectively be stated in terms of confidence interval providing a range for the prediction based on an acceptable uncertainty. Application of modeling and simulation techniques is well known in modern research of concrete materials [16]-[17].

The discussion of the literature reveals that statistical approach for analyzing strength and other properties of concrete results in meaningful interpretation and development of appropriate correlations for prediction of hardened properties. Application of existing statistical methodology has significant potential for interpretation of test results if systematic approach is adopted [18]–[19]. The two most fundamental aspects of experimental test results are adequate sample size and distribution characteristics. Most of the experiments deal with low sample size due to economic and time restrictions. Prediction of any correlation from this low sample size results in unreliable predicted values. Hence determination of adequate sample size or generation of adequate sample size from the available low sample size is a necessity. Any statistical approach dealing with sample size must take into account the distribution characteristics of data points. This fundamental problem is required to be addressed through a systematic statistical methodology along with conventional statistics. This paper is focused on these aspects of test results and the approach is described in reference to experimental test results (compressive strength) of self compacting concrete and normally compacting concrete.

II. MATERIAL

Ordinary Portland cement of 43 grade was used throughout the course of the investigation. The physical property of the cement conformed to Indian Standard code of practice [20]. A low calcium fly ash obtained from combined fields of the electrostatic precipitator of a thermal power plant was used. The 45 micron passing fraction in the unprocessed fly ash was more than 90 percent. Micro silica of grade 920 U with silica content of more than 92 percent was used.

The normally compacting concrete (NCC) had a cement content of 490 kg/m^3 with water cement ratio of 0.35. A fine aggregate content of 690 kg/m^3 and a total coarse aggregate content of 1000.6 kg/m^3 were adopted for the mix. A super-plasticizer dose of 2 kg/m^3 was used.

In the mix of self compacting concrete (SCC) a total powder content of 582.4 kg/m^3 was used, 50 percent of which was cement content and the remaining portion was flyash content. The mix had a fine aggregate content of 1062.4 kg/m^3 against a total coarse aggregate content of 455 kg/m^3. A limited amount of silica fume (14.6 kg/m^3) was mixed in the mix which had a water powder ratio of 0.32. A poly-carboxylic ether based super plasticizer of 1.67 percent of total powder content was applied to get the desired flow ability of SCC mix.

The compressive strength test was performed on 150 mm standard size cubes after the desired curing period. The specimens were demoulded after 24 hours of casting and were placed in a fixed temperature tank at 27 ± 2^0 C. The specimens were removed from water at 7, 28, 60, 90, 120 & 150 days and were tested in surface dried condition.

III. RESEARCH SIGNIFICANCE

There is considerable difference in the strength development pattern of normally compacting concrete and self compacting concrete due to the difference in mix composition and microstructure of the concretes [21]. Although a considerable body of research is available to investigate the variation of strength with age, the comparative analysis is primarily limited to graphical representation and age wise percentage calculation of strength gain [22]. Thus prevalent analysis based on low sample size provides little scope to understand the intrinsic details of the

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strength development due to associated unreliability of performance parameters derived from low sample size. The present research explores the statistical characteristics based on expanded sample size for comparative evaluation of compressive strength of SCC and NCC. The problem of determining the minimum sample size is addressed in this paper. A method is proposed to derive a minimum sample size from a new Root Mean Squared Spacing concept. This paper also focuses on distribution pattern (CDF analysis), dispersion of results (standard deviation and root mean squared spacing), relative comparison (ANOVA, ANOVA of dispersion) and signal to noise ratio. The parameters can be suitably used as control parameters for quality assurance.

IV. CONVENTIONAL STATISTICS

Analysis of variance

The analysis of variance method deals with multiple sample averages. It is a statistical hypothesis test with desired risk factor for comparison of multiple distinct data sets. A confidence level of 95% was adopted for the present experimental programme. The least squares approach is adopted in the ANOVA method. The ANOVA method is applied to the test results of normally compacting concrete (NCC) and self compacting concrete (SCC) for different curing periods. The F-ratio highlights existence of any significant difference of sample averages of two sets of test results for different curing ages and concretes. In the present analysis ANOVA provides significant insight into the comparative strength development pattern of NCC and SCC.

Signal to noise ratio

Signal to noise ratio is a measure of variation present in a set of test results. By evaluating the amount of variation present in a response quantity, factors contributing to variation can be diagnosed. Thus control factors can be identified for further improvement of quality. For the chosen response factor (compressive strength) the “higher is better” criteria is applied for calculation of S/N ratios following equation (1)

$$S/N = -10 \log \left(\frac{1}{n} \sum_{i=1}^{n} \frac{1}{y_i^2} \right) \quad (1)$$

Where n is the number of samples and yi is the value of compressive strength of the i th sample.

Monte Carlo Simulation

In any physical experimentation the availability of test samples is limited. This is due to the inaccessibility of test location or the robustness of the testing procedure or the cost involved. Success of any statistical procedure depends on the adequate sample size without which the output of any statistical process is unreliable. Thus a practical way to apply any statistical procedure on a set of test results with low sample size is to expand the set for generating adequate sample size without compromising basic physical properties of the test results. A suitable approach is the expansion of the set of test results to larger sample size following the experimental distribution pattern. The Monte Carlo simulation technique is employed here to generate simulated data of higher sample size for further statistical analysis. The simulation technique adopted does not require the knowledge of the type of distribution followed by the experiment results. But the simulated data set follows the same distribution pattern (known or unknown) as that of the test results. This can be achieved by inverse transformation of uniform random variants on the experimental numerical cumulative probability density curve. The simulated test results follow the same probability distribution as that of the experimental distribution. Statistical analysis performed on this simulated results provides reliable performance parameters.

V. PROPOSED METHOD

Root Mean Squared Spacing (RMSS) – A New Concept

An important statistical parameter to quantify dispersion of test results is the standard deviation. Standard deviation is evaluated around the mean value of sample test results following Equation (2).

Nσ2=∑(xi-μ)2 (2)

It is understood that the compressive strength of concrete is a random variable and the sequence of occurrence of the data points is important especially when low sample size is considered. The value of standard deviation signifies the overall dispersion of test results independent of the random occurrence of experimental data points.

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Thus any alternative arrangement of the same data points yields the same standard deviation. Moreover if the number and range of data increases with similar spacing of data points standard deviation increases accordingly with no indication of the spacing of data points. Thus an alternative parameter called Root Mean Squared Spacing is proposed here to incorporate the sequence effect of randomly distributed test results (compressive strength).

Procedure

A particular set of test results follow a definite distribution pattern which may fall in the known categories of distribution or may follow any unknown distribution type. For a large sample size the results are randomly sequenced according to the distribution of the data set. Hence occurrence of any data after a selected data point is purely random in nature and can be best described by a random vector. Summation of all such random vectors provides the collective random vector for the data set. Appling the principles of vector algebra and the assumption of large sample size results in a new parameter called Root Mean Squared Spacing. The new parameter (RMSS) is calculated using equation (3).

$$N\,(RMSS)^2 = \sum (x_{i+1} - x_i)^2 \qquad (3)$$

The right hand side of equation (3) depends on the sequence of the random results. The term $\sum(x_{i+1}-x_i)^2$ signifies the RMS difference after N random selections. For a large sample size of random test results $[\sum(x_{i+1}-x_i)]^2$ converges with $\sum(x_{i+1}-x_i)^2$. This can be proved using the random walk principle for randomly distributed test results. RMSS is used as a parameter in this paper for relative comparison of strength development pattern of SCC and NCC.

The physical difference between standard deviation and RMSS can be best understood through application on natural numbers. For a set of natural numbers the standard deviation increases with increase in the range of the set. A set of natural numbers from 1 to 100 has higher standard deviation than a set of numbers from 1 to 50 whereas but for both the sets the RMSS values are the same implying equal root mean squared spacing of the data points.

Problem definition

Any statistical analysis of performance parameters demands a minimum sample size for reliable output. Determination of a minimum sample size is a necessity during trial experimentation. Trial experimentation usually involves testing of a few samples instead of a large number of samples of statistical significance. Hence there is a necessity to determine the minimum sample size of statistical dimension from a set of trial test results of low sample size. The concept of minimum sample size will restrict the experimentation to a reasonable level to make it cost effective.

VI. APPROACH TO SOLUTION

Assumptions

The solution methodology is based on the basic assumption of large sample size and random distribution of test results and simulated results. The assumption imposes cancellation of symmetric terms in the expression of vector algebra resulting in simplification of statistical expressions.

Necessity of RMSS concept

The basic unreliability of statistical parameters of low sample size experimentation is caused due to the sequence effect of randomly distributed data. The sequence effect may be understood by the change in values of any statistical parameters with the change in the sequence of placement of the data points of any distribution pattern. The minimum sample size may thus be defined as a sample size for which the sequence effect is negligible. The standard deviation does not include any term for sequence effect. Hence the RMSS parameter is considered to include the sequence effect as described in the previous sub section.

VII. SOLUTION METHODOLOGY

Determination of adequate sample size using RMSS concept-A proposed method

It is interesting to note that RMSS value is higher than the standard deviations in all the cases. But the RMSS value is not in increasing functional relationship with standard deviation. Mathematical derivation based on the

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assumption of large sample size and random sequencing of the data results in the following relationship between RMSS and standard deviation.

$$RMSS=\sigma\sqrt{2}\left[\sqrt{\frac{1-\sum_{i=1}^{n-1} x_i x_{i+1}/s}{1-n\mu^2/s}}\right] \quad (4)$$

Where σ is the standard deviation of the test results. xi and xi+1 are the i th and (i+1) th test result in random sequencing. "s" is the sum of squares of test results. n is the number of samples and μ is the mean value. The term in the parenthesis provides the sequence effect and is nearly equal to 1 for the large data as generated and sequenced through Monte Carlo simulation. RMSS/ σ√2 is the sequence co-efficient and limits towards 1 for large sample size. Hence RMSS can be taken approximately equal to √2 times the standard deviation for a considerably large sample size. Thus RMSS can be used as a quality control parameter similar to the standard deviation for large sample size.

The term in the parenthesis of equation 4 is called the sequence co-efficient (α). For large sample size α tends towards 1 in random sequencing. The valid question arises now is "How large a sample size should be to call it large enough?" Equating the sequence co-efficient α to 1 provides the following condition for large random data.

$$\frac{\sum_{i=1}^{n-1} x_i x_{i+1}}{\left[\frac{\left(\sum x_i\right)^2}{n}\right]} \approx 1 \quad (5)$$

Thus a sample size can be said as large enough for the given set of test results when the condition of equation 5 is closely satisfied. The minimum sample size for a given test condition can thus be evaluated from a few trial experimentation and applying the condition of equation 5. In the present test cases Monte Carlo simulation can generate a large number of simulated data from the limited number of trial samples (ten in the present case). Going by the condition of equation 5 a considerably large enough sample size can be determined when the ratio of the left side of equation (5) is nearly equal to 1. This provides a procedure for determining adequate sample size from a few trial experimentations using Monte Carlo simulation technique. Analysis shows that a sample size around 50 provides adequate test results of statistical significance. In other words the sample size can be considered adequate when (RMSS/ σ) approaches √2.

The procedure can also be applied to design experimentation for more reliable test results. The trial experimentation with a few samples can be expanded through Monte Carlo simulation technique. The required minimum sample size can be determined using the RMSS concept as described here. Experimentation with this minimum sample size will result in more accurate and reliable test results.

VIII. Result and Discussion

Expansion of data

Compressive strength test for NCC and SCC were conducted at different ages with ten samples for each case. This was a low sample size and required a larger sample size for any significant statistical analysis. Such a situation may arise in any practical experimentation due to inability of robust experimentation of statistical magnitude. A practical means to address such a problem is the expansion of the data set without compromising the essential physical attributes. The test results of compressive strength of NCC and SCC are given in Tables 1 and 2.

Table 1: Compressive Strength of Normally Compacting Concrete (NCC).

Specimen No	Compressive strength (N/mm2)					
	Age (days)					
	7	28	60	90	120	150

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1	40.0	58.0	61.5	64.0	64.0	67.0
2	41.0	62.0	60.5	62.5	60.0	62.0
3	38.0	60.0	58.0	66.0	67.3	64.0
4	40.0	55.0	65.5	67.5	64.0	60.0
5	33.0	58.5	58.5	60.0	57.0	65.0
6	40.0	61.5	64.0	58.0	63.5	68.0
7	42.0	59.0	57.0	63.7	59.0	58.0
8	36.0	54.5	64.0	58.2	64.0	62.0
9	38.5	63.0	61.5	62.0	62.0	62.0
10	41.5	56.0	58.5	63.0	67.0	62.0

Table 2: Compressive Strength of Self Compacting Concrete (SCC).

Specimen No	Compressive strength (N/mm2)					
	Age (days)					
	7	28	60	90	120	150
1	30.0	46.5	55.0	63.0	62.0	62.0
2	25.0	48.0	58.0	55.0	55.0	66.0
3	22.0	45.0	52.0	63.0	66.0	60.0
4	35.0	56.0	50.0	58.0	60.0	60.0
5	28.0	40.0	60.0	54.0	60.0	65.0
6	30.0	45.0	62.0	57.0	58.0	55.0
7	32.0	55.0	48.0	60.0	57.0	60.0
8	25.0	40.0	46.0	64.0	60.0	55.0
9	35.0	41.5	54.0	52.0	54.0	67.5
10	28.0	46.0	58.0	53.0	68.0	65.5

The empirical cumulative distribution plot is done after refinement of the data by exclusion of the repetition values. Further the set of ten results is expanded using Monte Carlo simulation technique to generate hundred results Table 3.

Table 3: Comparison of Mean and Standard Deviation of Test Data and Simulated Data.

Identification	Test Data		Simulated Data	
	Mean	Standard Deviation	Mean	Standard Deviation
7 day SCC	28.67	4.72	26.94	4.17
28 day SCC	47.25	5.73	47.12	5.03
60 day SCC	53.89	5.44	52.22	4.89
90 day SCC	57.33	4.30	57.02	3.92
120 day SCC	60.00	5.04	58.72	3.84
150 day SCC	63.00	4.35	62.05	5.66
7 day NCC	38.75	3.07	37.71	3.40
28 day NCC	58.75	2.95	58.52	2.79
60 day NCC	60.71	3.17	59.36	3.11
90 day NCC	62.49	3.10	62.37	2.63
120 day NCC	62.48	3.69	61.03	4.00
150 day NCC	63.43	3.65	62.98	3.39

The empirical CDF plots of the experimental results are depicted in Figures 1, 2, 3 and 4.

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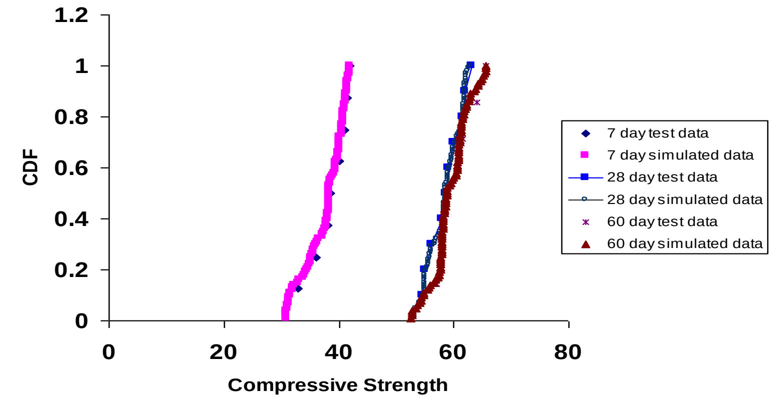

Fig 1:Comparison between Test Data and Simulated Data for NCC

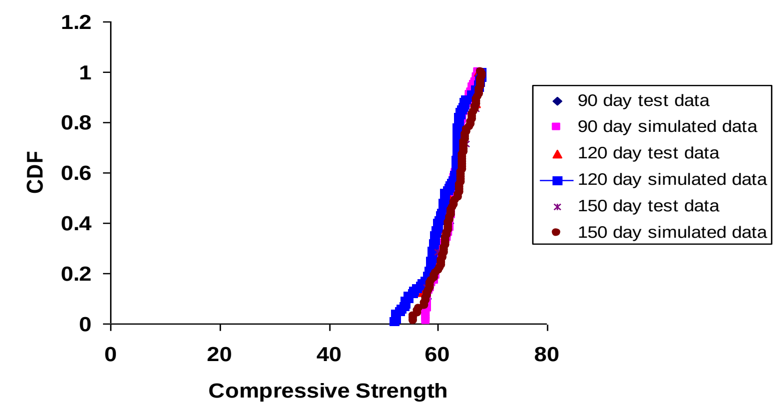

Fig 2:Comparison between Test Data and Simulated Data for NCC

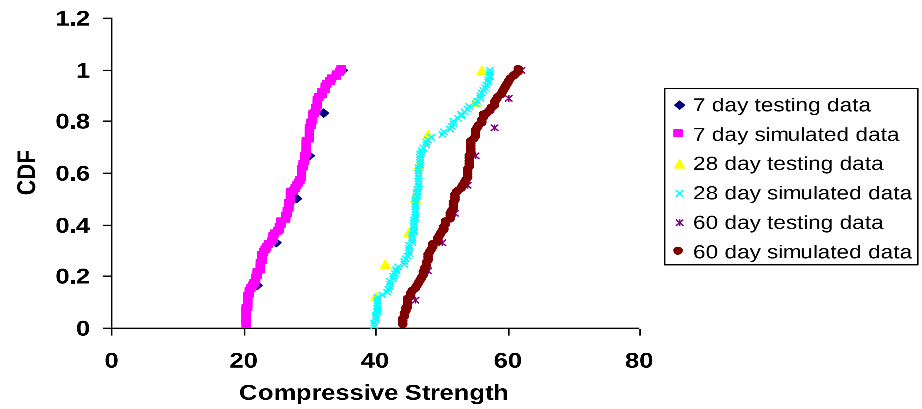

Fig 3:Comparison between Test Data and Simulated Data for SCC

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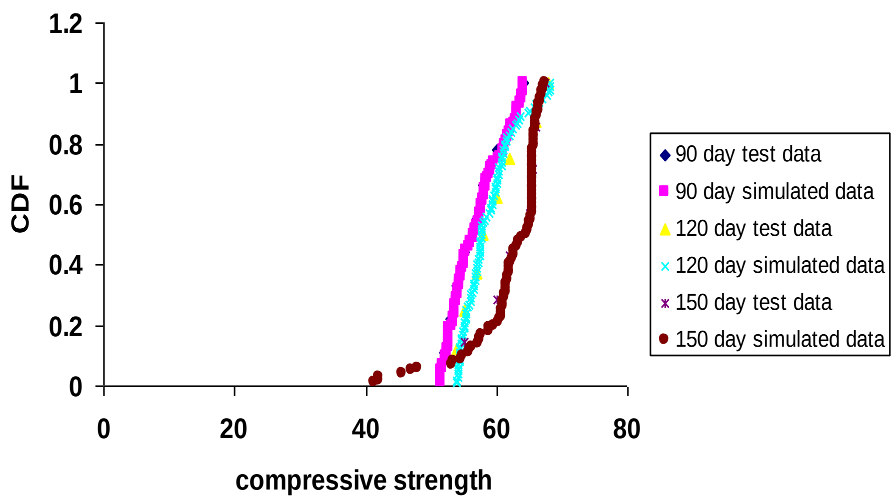

Fig 4:Comparison between Test Data and Simulated Data for SCC

Cumulative distribution of test results and simulated results match closely in all the cases. The mean and standard deviation of test results and simulated results have negligible difference and justifies the principle of simulation Table 3. Hence detailed statistical analysis with simulated test results is expected to represent the characteristics of the actual test results.

Analysis of variance

ANOVA provides very basic parameter for statistical comparison of two different data sets. ANOVA analysis reveals that compressive strength of SCC and NCC have significant difference for 7,28,60 and 90 days age with the highest F-ratio for 7 and 28 days Table 4 and 5 but for 120 and 150 days age F-ratio fall below the critical F value indicating similar strength pattern for higher ages.

Table 4: ANOVA Analysis between NCC and SCC with Test Data of Same Ages.

ANOVA Between	Calculated F value from ANOVA	Critical F value
7 day NCC and 7 day SCC	38.38	4.41
28 day NCC and 28 day SCC	39.11	4.41
60 day NCC and 60 day SCC	11.93	4.41
90 day NCC and 90 day SCC	7.20	4.41
120 day NCC and 120 day SCC	2.51	4.41
150 day NCC and 150 day SCC	0.67	4.41

Table 5: ANOVA Analysis between NCC and SCC with Simulated Data of Same Ages.

ANOVA Between	Calculated F value from ANOVA	Critical F value
7 day NCC and 7 day SCC	400.78	3.89
28 day NCC and 28 day SCC	392.58	3.89
60 day NCC and 60 day SCC	151.56	3.89
90 day NCC and 90 day SCC	128.46	3.89
120 day NCC and 120 day SCC	17.23	3.89
150 day NCC and 150 day SCC	2.02	3.89

The F-ratios are graphically presented in Figure 5 for experimental and simulated results.

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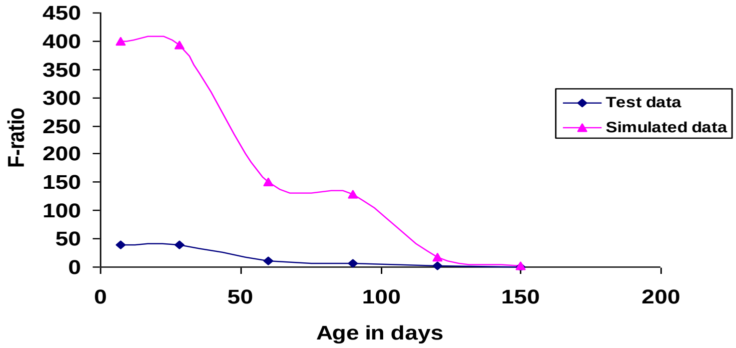

Fig 5: F- ratio between SCC and NCC for Different Ages

The fall of F-ratio from the highest point at 7days to the lowest point at 150 days depicts the fall in the difference of strength pattern of SCC and NCC at higher ages. Another set of ANOVA analysis is performed to understand the effect of curing age on strength development Tables 6, 7, 8 and 9.

TABLE 6: ANOVA ANALYSIS OF TEST DATA OF NCC FOR DIFFERENT AGES.

ANOVA BETWEEN	CALCULATED F VALUE FROM ANOVA	CRITICAL F VALUE
7 day NCC and 150 day NCC	339.93	4.41
28 day NCC and 150 day NCC	10.03	4.41
60 day NCC and 150 day NCC	2.47	4.41
90 day NCC and 150 day NCC	0.14	4.41
120 day NCC and 150 day NCC	0.02	4.41
150 day NCC and 150 day NCC	0	4.41

TABLE 7: ANOVA ANALYSIS OF SIMULATED DATA OF NCC FOR DIFFERENT AGES.

ANOVA BETWEEN	CALCULATED F VALUE FROM ANOVA	CRITICAL F VALUE
7 day NCC and 150 day NCC	2770.21	3.89
28 day NCC and 150 day NCC	103.25	3.89
60 day NCC and 150 day NCC	62.14	3.89
90 day NCC and 150 day NCC	2.05	3.89
120 day NCC and 150 day NCC	13.96	3.89
150 day NCC and 150 day NCC	0	3.89

TABLE 8: ANOVA ANALYSIS OF TEST DATA OF SCC FOR DIFFERENT AGES.

ANOVA BETWEEN	CALCULATED F VALUE FROM ANOVA	CRITICAL F VALUE
7 day SCC and 150 day SCC	279.76	4.41
28 day SCC and 150 day SCC	46.35	4.41
60 day SCC and 150 day SCC	11.21	4.41
90 day SCC and 150 day SCC	3.49	4.41
120 day SCC and 150 day SCC	0.65	4.41
150 day SCC and 150 day SCC	0	4.41

TABLE 9: ANOVA ANALYSIS OF SIMULATED DATA OF SCC FOR DIFFERENT AGES.

ANOVA BETWEEN	CALCULATED F VALUE FROM ANOVA	CRITICAL F VALUE
7 day SCC and 150 day SCC	2496.27	3.89
28 day SCC and 150 day SCC	388.85	3.89

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60 day SCC and 150 day SCC	172.75	3.89
90 day SCC and 150 day SCC	53.40	3.89
120 day SCC and 150 day SCC	23.65	3.89
150 day SCC and 150 day SCC	0	3.89

150 days strength is taken as the ultimate standard strength pattern as there is non significant strength gain beyond this curing period. It is interesting that value of F-ratio between 150 days and 150 days comes down to zero for both the concretes. Thus relative change of F ratio with age indicates relative rise or fall of the multiple sample averages for the respective ages. There is rapid fall of F-ratios from 7 days to 28 days indicating significant change in strength during this period Figure 6.

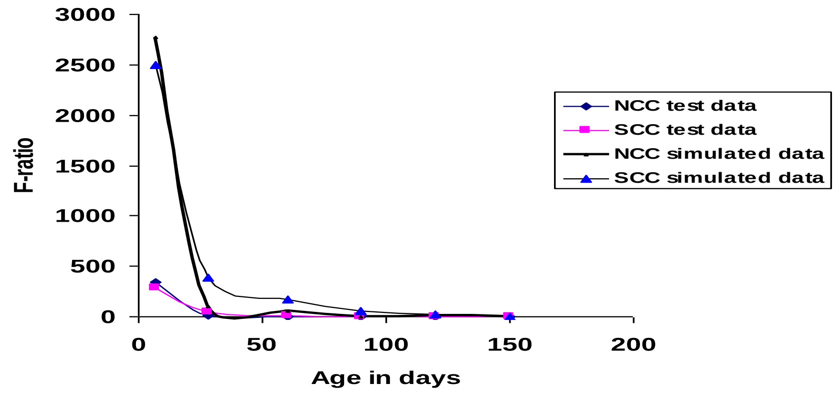

Fig 6: F-ratio of NCC and SCC for 150 Days Age

The critical F-ratio value serves as a datum to estimate the level of relative significance of the strength level at any age with respect to the standard result.

Signal to noise ratio

Signal to noise ratio is an indication of the measure of variation present and also is an estimation of unwanted influence (noise) present in the test results. Higher signal to noise ratio is more desirable for any test results. It is found from Table 3 that standard deviation of test result has no uniform correlation with the age of concrete. As age increases standard deviation of test results or simulated results may increase or decrease with no indication of any understandable pattern. Thus only standard deviation is not a complete parameter to assess the convergence of different influence on strength pattern. In contrast, Signal to Noise ratio increases with curing age from 7 days to 150 days for NCC and SCC considering both test results and simulated results Table 10.

TABLE 10: S/N RATIOS OF SCC AND NCC AT DIFFERENT AGES.

AGE OF CONCRETE (DAYS)	NORMALLY COMPACTING CONCRETE		SELF COMPACTING CONCRETE	
	TEST DATA	SIMULATED DATA	TEST DATA	SIMULATED DATA
7	31.76	28.98	31.41	28.28
28	35.35	33.15	35.32	33.33
60	35.67	34.58	35.43	34.24
90	35.89	35.18	35.88	35.06
120	35.92	35.50	35.65	35.33
150	35.96	35.73	35.95	35.71

Also the difference of S/N ratios between 7 days and 28 days is much higher than the difference of the same between any other ages. It indicates that upto 28 days for NCC and 60 days for SCC the unwanted influence is considerably higher. For 150 days strength S/N ratio is the highest for both NCC and SCC revealing convergence of influences at higher ages. It is also found that S/N ratios of experimental results and simulated results are nearly the same and is almost equal at 150 days. Thus signal to noise ratio provides a better estimate of variation than standard deviation due to the incorporation of different influences (noise) in the concept of S/N ratio. Hence in S/N

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ratio has better significance than change in standard deviation to understand strength development pattern of concrete.

Root Mean Squared Spacing

Root mean squared spacing (RMSS) is a new parameter introduced in this analysis primarily to take into account the sequence effect of randomly distributed data as obtained by Monte Carlo simulation. The basic difference between RMSS and standard deviation is discussed in the previous article. The RMSS values are shown in Table11.

Table 11: RMSS and Standard Deviation of SCC and NCC at Different Ages.

Age of concrete (days)	Normally compacting concrete		Self compacting concrete	
	Standard deviation	RMSS	Standard deviation	RMSS
7	3.39	4.65	4.17	5.75
28	2.79	3.78	5.03	6.08
60	3.11	4.11	4.89	6.66
90	2.63	3.60	3.92	4.91
120	4.00	5.40	3.84	5.30
150	3.39	4.90	5.66	8.91

The sequence coefficient for concrete strength of SCC and NCC at 150 days is shown in Table 12.

Table 12: Sequence co-efficient (a) for NCC and SCC at 150 Days.

Sample size	NCC	SCC
10	0.893805	0.88953
20	0.944103	0.943913
30	0.966097	0.965301
40	0.972256	0.970878
50	0.979517	0.97875
60	0.981504	0.980253
70	0.984515	0.983776
80	0.986298	0.985812
90	0.987635	0.987425
100	0.988234	0.988072

ANOVA analysis of standard deviation and RMSS

To understand the dispersion of test results standard deviation and RMSS are well applicable. To analyze whether there is substantial difference between dispersion of test results of different concretes ANOVA of the dispersion parameters provides F-ratio for further comparison of dispersion effect between the concretes. It is found that the F-ratio of standard deviation between NCC and SCC (14.63) is considerably higher than the critical F-ratio (4.96), see Table13. Similar observation is found for RMSS also Table 14.

Table 13: ANOVA Analysis of Standard Deviation of NCC and SCC.

FACTOR	SOURCE OF VARIATION	SUM OF SQUARE	DEGREE OF FREEDOM	MEAN SQUARE	F-VALUE	FCRI
NCC	BETWEEN FACTOR	5.60	1	5.60	14.64	4.96 ($F_{0.05,1,10}$)
SCC	WITHIN FACTOR	3.83	10	0.38		
	TOTAL	9.43	11	5.98		

Table 14: ANOVA Analysis of RMSS Value of NCC and SCC.

FACTOR	SOURCE OF VARIATION	SUM OF SQUARE	DEGREE OF FREEDOM	MEAN SQUARE	F-VALUE	FCRI
NCC	BETWEEN	10.39	1	10.39	8.21	4.96

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	FACTOR					($F_{0.05,1,10}$)
SCC	WITHIN FACTOR	12.65	10	1.26		
	TOTAL	23.05	11	11.66		

IX. CONCLUSION

A new concept is proposed to estimate adequate sample size of statistical significance and a new parameter, Sequence coefficient, is developed to estimate the sample size. It is observed that sample size can be called adequate if the sequence coefficient approaches 1 or (RMSS/ σ) approaches √2. The new method has further potential to design more accurate testing programme for better test results. The method can be tested further before widespread application in scientific experimentation.

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