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Forced Flow of a Newtonian Fluid Due to a Vertical Stretching Sheet with Viscous Dissipation

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Abstract*— Similarity transformation is used to convert the governing non-linear partial differential equations into non linear ordinary differential equations. The two- point boundary value problem arising in the study is solved using the differential transform method combined with rational approximation. Convergence of the infinite power series is checked in the computation process for each parameter's combination and similarly the decision regarding infinity is made. Assistance is taken from the Shooting method based on RK45 and Newton-Raphson method in specifying the unknown initial values. The component of the streamline and the temperature distribution are found to be decreasing functions of the transverse coordinate, the effect of Buoyancy parameter, Suction parameter, Prandtl number, Radiation parameter, Eckert number and stream function. These profiles are clearly presented through figures. The present study has applications in fluid dynamical problems involving thin films.

Index Terms— **Boundary layer, Buoyancy parameter, Suction parameter, Prandtl number, Radiation parameter, Eckert number, heat transfer, Differential transform method, Padé approximant,** Shooting **method.**

I. INTRODUCTION

Boundary layer flow and heat transfer over a linearly stretched surface has received considerable attention in recent years, because of its various applications in engineering and metallurgical problems such as hot rolling, wire drawing, metal and plastic extrusion, continuous casting, glass fiber production, crystal growing, paper production. Sakiadis [1] was the first person to study about boundary layer flow over a stretched surface moving with a constant velocity. Erickson et al.[2] extended the work of Sakiadis [1] and later Chen and Char [3], Elbashbeshy[4] investigated the effects of variable surface temperature and heat flux on the heat transfer characteristics of a linearly stretched sheet subject to blowing or suction. The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and these concerned with dissociating fluids. Possible heat generation effects may affect the temperature distribution consequently, the particle deposition rate in nuclear reactors, electronic chips and semi conductor wafers. Vajravelu and Hadjinicalaou [5] studied the heat transfer characteristics over a stretching surface with viscous dissipation in the presence of internal heat generation or absorption.

The problem of natural convection along a vertical isothermal or uniform flux plate is a classical problem. However, Gebhart [6] studied the problem by considering viscous dissipation. Recently, Copiello and Fabbri [7] studied the effect of viscous dissipation on the heat transfer in sinusoidal profile finned dissipaters, Alam et al. [8] considered the effect of viscous dissipation in natural convection over a sphere. Pantokratoras [9] studied the effect of viscous dissipation in natural convection in a new way. However, in this work we have investigated the combined effect of thermal radiation, heat generation and viscous dissipation on steady free convection heat transfer flow over a stretching sheet.

II. MATHEMATICAL FORMULATION

Major headings should be typeset in boldface with the first letter of important words capitalized. A steady two dimensional free convection laminar boundary layer flow of a viscous incompressible fluid along a stretching sheet with heat generation under the influence of thermal radiation and viscous dissipation is considered in (Fig.1). The x-axis is taken along the stretching sheet in the vertically upward direction and the y-axis is taken as normal to the sheet. Two equal and opposite forces are introduced along the x-axis, so that the sheet is stretched keeping the origin fixed. The plate is maintained at a constant temperature T_w and the ambient temperature of the flow is T_∞. The fluid is considered to be gray, absorbing – emitting radiation but non- scattering medium and the Rosseland approximation is used to describe the radioactive heat flux in the energy equation. The radioactive heat flux in the x-direction is considered negligible in comparison to the y-direction.

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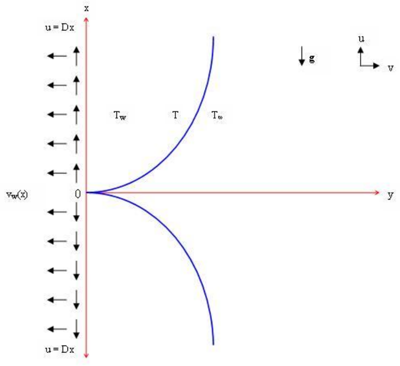

Fig. 1 Sketch of the physical model and coordinate system

III. GOVERNING EQUATIONS

Under the above assumption, the governing boundary layer equations are:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty). \tag{2}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2. \tag{3}$$

The appropriate boundary conditions are:

$$\left.\begin{aligned} u=Dx,\quad v=v_w,\quad T=T_w,\quad \text{at}\ \ y=0\\ u=0,\quad T=T_\infty\quad \text{as}\ \ y\to\infty\end{aligned}\right\}, \tag{4}$$

where u and v are the velocity components in the x and y directions respectively. ν is the kinematic viscosity, g is the acceleration due to gravity. T is the fluid temperature in the boundary layer, ρ is the density of the fluid, k is the thermal conductivity of the fluid , c_p is the specific heat at constant pressure, q_r is the radiative heat flux, D(>0) is the stretching constant and v_w is the velocity component at the wall having positive value to indicate suction.

The radiative heat flux q_r is described by the Rosseland approximation such that:

$$q_r=\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ_1 is the Stefan-Boltzman constant and k_1 is the Rosseland mean absorption coefficient. Following Chamkha [11], it is assumed that the temperature difference within the flow and sufficiently small such that T^4 can be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about the free stream temperature T_∞ and neglecting the higher order terms. This results in the following approximation:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4, \tag{6}$$

using (5) and (6) in the last term of equation (3), we obtain:

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma_1 T_\infty^3}{3k_1}\frac{\partial^2 T}{\partial y^2}, \tag{7}$$

interchanging in equation (3), we obtain the following energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{k}{\rho c_p} + \frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1}\right)\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 . \tag{8}$$

IV. NON-DIMENSIONLISATION

We introduce the stream function ψ related to u and v as defined below:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x} . \tag{9}$$

Introducing (9) in eq. (2) and (8), we get

$$\left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y}\right)\frac{\partial^2 \psi}{\partial x \partial y} = \nu \frac{\partial^3 \psi}{\partial y^3} + g\beta(T - T_\infty), \tag{10}$$

$$-\frac{\partial(\psi, T)}{\partial(x, y)} = \left(\frac{k}{\rho c_p} + \frac{16\sigma_1 T_\infty^3}{3\rho c_p k_1}\right)\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial^3 \psi}{\partial y^3}\right)^2 . \tag{11}$$

In order to obtain a similarity solution of the problem, we introduce a similarity parameter δ such that δ is length scale. We now introduce the following dimensionless variables:

$$X = x\sqrt{\frac{D}{\nu}},\quad Y = \frac{y}{\delta} = y\sqrt{\frac{D}{\nu}},\quad \psi(x,y) = \sqrt{D\nu x}\ f(Y), \Psi = \frac{\psi}{\sqrt{D\nu}\left(\frac{\nu}{D}\right)^{1/4}} = \sqrt{X}\, f(Y),\quad \theta(Y) = \frac{T - T_\infty}{T_w - T_\infty} \tag{12}$$

Using equation (12) in equations (10) and (11), we obtain the following dimensionless equations:

$$f''' + f f'' - (f')^2 + \lambda\theta = 0 , \tag{13}$$

$$\theta'' + \frac{3N}{3N+4}\Pr f\theta' + \frac{3N}{3N+4}\Pr Ec (f'')^2 = 0 , \tag{14}$$

where,

$$\lambda = \frac{Gr}{Re^2} = \frac{g\beta(T_w - T_\infty)}{D^2 x} \text{ (buoyancy parameter)},$$

$$\Pr = \frac{\mu c_p}{k} \text{ (Prandtl number)},\quad N = \frac{k k_1}{4\sigma_1 T_\infty^3} \text{ (radiation parameter)},\quad Ec = \frac{D^2 x^2}{c_p(T_w - T_\infty)} \text{ (Eckert number)}.$$

The transformed boundary conditions are:

$$\left.\begin{aligned} f' = 1,\quad f = F_w,\quad \theta = 1 \quad \text{at } Y = 0 \\ f' = 0,\quad \theta = 0 \quad \text{as } Y \to \infty \end{aligned}\right\} \tag{15}$$

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where, $F_w = -\frac{v_w}{\sqrt{Dv}}$ is the suction parameter.

In the next sections we propose the differential transform method coupled with Padé approximation for solving the nonlinear boundary value problem in equations (13) – (15).The initial unknown values are obtained using shooting method based on RK45 and Newton-Raphson method.

V. EQUIVALENT IVP AND DIFFERENTIAL TRANSFORM METHOD OF SOLUTION

The BVPs in equation (9)-(12) is written as an equivalent initial value problem as follows:

$$f''' + f f'' - (f')^2 + \lambda\theta = 0 \ , \qquad (16)$$

$$\theta'' + \frac{3N}{3N+4} \Pr f \theta' + \frac{3N}{3N+4} \Pr Ec (f'')^2 = 0 \ , \qquad (17)$$

with initial conditions:

$$\left.\begin{aligned} & f = Fw,\ f' = 1,\ f'' = \alpha \ \text{ at } Y = 0, \\ & \theta = 1\ \ \theta' = \beta \ \text{ at } Y = 0 \end{aligned}\right\} \qquad (18)$$

where α, β are determined from the shooting method .

Table1: Values of α, β for buoyancy parameter (λ) =3.0, Prandtl number (Pr) = 0.71, Radiation parameter (N) = 1.0, Eckert number (Ec) = 3.0 and different values of suction parameter (Fw)

Suction parameter (Fw)	α	β
1.0	0.72822714	-0.35159372
2.0	0.13079388	-0.57463110
3.0	-0.82179229	-0.65872533
5.0	-3.18711613	-0.39002414
10.0	-8.94193868	0.75516445

Table2: Values of α, β for suction parameter (Fw) =2.0, Prandtl number (Pr) = 0.71, radiation parameter (N) = 0.3, Eckert number (Ec) = 1.0 and different values of buoyancy parameter (λ)

Buoyancy parameter (λ)	α	β
1.0	-1.08273316	-0.33478565
2.0	-0.17009000	-0.38782612
3.0	0.62790334	-0.40171328
5.0	2.06909375	-0.36198469
7.0	3.41488091	-0.25741463

Table3: Values of α, β for suction parameter (Fw) =2.0, buoyancy parameter (λ) =7.0, radiation parameter (N) = 1.0, Eckert number (Ec) = 2.0 and different values of Prandtl number (Pr)

Prandtl number (Pr)	α	β
0.71	3.03316442	0.02567490
1.0	2.67796559	-0.07731300
2.0	1.80351502	-0.66531094
3.0	1.21517988	-1.32464609
7.0	0.21004376	-3.16916288

Table 4: Values of α, β for suction parameter (Fw) =3.0, buoyancy parameter (λ) =7.0, Prandtl number (Pr)= 0.71, Eckert number (Ec) = 2.0 and different values of radiation parameter (N)

Radiation parameter (N)	α	β
0.2	4.03927906	-0.07392148
0.5	2.80794469	-0.33301810

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1.0	1.95112363	-0.64826299
2.0	1.24135822	-0.98019402
5.0	0.67250926	-1.28254736

Table 5: Values of α, β for Suction parameter (Fw) =2.0, Buoyancy parameter (λ) = 4.0, Prandtl number (Pr) = 0.71, Radiation parameter (N) = 1.0 and different values of Eckert number (Ec)

Eckert number (Ec)	α	β
0.5	0.43094440	-0.78188795
1.0	0.51801340	-0.73435332
2.0	0.70484986	-0.62970141
3.0	0.91782507	-0.49583403
4.0	1.17761785	-0.29587406

Using the DTM Zohu [12] we obtain the solution of f, θ in the following manner.
Transformation of the k^{th} derivative of a function in one variable is defined as follows:

$$F[k] = \frac{1}{k!}\left[\frac{d^k f(Y)}{dY^k}\right]_{Y=Y_0} \quad (19)$$

and the inverse transformation is defined by

$$f(Y) = \sum_{k=0}^{\infty} F[k]\ (Y - Y_0)^k \quad (20)$$

where F[k] is the differential transform of f(Y).
Taking differential transform of equations (16) and (17), we have:

$$(k+3)(k+2)(k+1)F[k+3] + \sum_{l=0}^{k} F[l](k+2-l)(k+1-l)F[k+2-l]$$
$$-\sum_{l=0}^{k}(l+1)F[l+1](k+1-l)F[k+1-l] + \lambda\theta[k] = 0, \quad (k = 0,1,2,...,30), \quad (21)$$

$$(k+2)(k+1)\theta[k+2] + \frac{3N}{3N+4}\Pr\sum_{l=0}^{k} F[l]\ (k+1-l)\theta[k+1-l]$$
$$+\frac{3N}{3N+4}\Pr Ec\sum_{l=0}^{k}(l+2)(l+1)F[l+2](k+2-l)(k+1-l)F[k+2-l] = 0, \ (k = 0,1,2,...,31), \quad (22)$$

where F[k] and $\theta[k]$ are the differential transform of $f(Y)$ and $\theta(Y)$. The transformed boundary conditions are

$$F[0] = F_w, \quad F[1] = 1, \quad F[2] = \frac{\alpha}{2},$$
$$\theta[0] = 1, \quad \theta[1] = \beta \quad (23)$$

From table 1, for $\lambda = 3.0, \Pr = 0.71, N = 1.0, Ec = 3.0, Fw = 5.0$,
we get $\alpha = -3.18711613; \ \beta = -0.39002414$
and the solutions of equations (16) & (17) using DTM as follows:

$f(Y) = 5 + Y - 1.59356\,Y^2 + 2.3226\,Y^3 - 2.98729\,Y^4 + 3.28885\,Y^5 - 3.15776\,Y^6 + 2.7631\,Y^7 - 2.32318\,Y^8 + 1.96155\,Y^9 - 1.69083\,Y^{10} + 1.47018\,Y^{11} - 1.26175\,Y^{12} + 1.05324\,Y^{13} - 0.852495\,Y^{14} + 0.673086\,Y^{15} - 0.523916\,Y^{16} + 0.406335\,Y^{17} - 0.316219$

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ISO 9001:2008 Certified
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Volume 2, Issue 4, July 2013
Y^{18} + 0.247394 Y^{19} - 0.194122 Y^{20} + 0.152127 Y^{21} - 0.118579 Y^{22} + 0.0916659 Y^{23} - 0.0701522 Y^{24} + 0.053103 Y^{25} - 0.0397453 Y^{26} + 0.0294157 Y^{27} - 0.0215405 Y^{28} + 0.0156264 Y^{29} - 0.0112522 Y^{30} + 0.00806269 Y^{31} - 0.00576428 Y^{32} + 0.00412103 Y^{33}, (24)

$\theta(Y)$ =1- 0.390024 Y - 4.33835 Y^2 + 15.7302 Y^3 -37.9236 Y^4 + 75.3432 Y^5 - 129.835 Y^6 + 198.673 Y^7 - 274.868 Y^8 + 349.745 Y^9 - 416.221Y^{10} + 470.777 Y^{11} - 513.182 Y^{12} + 544.709 Y^{13} -566.361 Y^{14} + 578.177 Y^{15} - 579.656 Y^{16} + 570.603 Y^{17} -551.697 Y^{18} + 524.5 Y^{19} - 491.073 Y^{20} + 453.514 Y^{21} - 413.659 Y^{22} + 372.999 Y^{23} - 332.713 Y^{24} + 293.745 Y^{25} - 256.842 Y^{26} + 222.558 Y^{27} - 191.247 Y^{28} + 163.073 Y^{29} - 138.037 Y^{30} + 116.019 Y^{31} - 96.8265 Y^{32} + 80.2285 Y^{33}, (25)

VI. PADÉ APPROXIMATION

Some techniques exist to increase the convergence radius of a given series. Among them, the so called Padé technique is widely applied. Suppose that a function $f(\eta)$ is represented by a power series $\sum_{i=0}^{\infty} c_i \eta^i$ so that

$$f(\eta) = \sum_{i=0}^{\infty} c_i \eta^i \qquad (26)$$

This expansion is the fundamental starting point of any analysis using Padé approximants. The notation c_i, i =0,1,2,…is reserved for the given set of coefficients and $f(\eta)$ is the associated function

$$\frac{a_0 + a_1\eta + \ldots + a_L \eta^L}{b_0 + b_1\eta + \ldots + b_M \eta^M} \qquad (27)$$

This has a Maclaurin expansion which agrees with equation (26) as far as possible. Notice that in equation (27) there are L+1 numerator coefficients and M+1 denominator coefficients (Baker [10]), So there are L+1 independent coefficients and M independent coefficients, making L+M+1 unknown coefficients in all. This number suggests that normally [L,M] ought to fit the power series equation (26) through the orders $1, \eta, \eta^2, \ldots, \eta^{L-M}$, in the notation of formal power series

$$\sum_{i=0}^{\infty} c_i \eta^i = \frac{a_0 + a_1\eta + \ldots + a_L \eta^L}{b_0 + b_1\eta + \ldots + b_M \eta^M} + O(\eta)^{L+M+1} \qquad (28)$$

Baker (1981) found that

$$(b_0 + b_1\eta + \ldots + b_M \eta^M)(c_0 + c_1\eta + \ldots) = a_0 + a_1\eta + \ldots + a_L \eta^L + O(\eta^{L+M+1}) \qquad (29)$$

Equating the coefficient of $\eta^{L+1}, \eta^{L+2}, \ldots, \eta^{L+M}$

$$\begin{aligned} b_M c_{L-M+1} + b_{M-1} c_{L-M+2} + \ldots + b_0 c_{L+1} &= 0 \\ b_M c_{L-M+2} + b_{M-1} c_{L-M+3} + \ldots + b_0 c_{L+2} &= 0 \\ &\vdots \\ b_M c_L + b_{M-1} c_{L+1} + \ldots + b_0 c_{L+M} &= 0 \end{aligned} \qquad (30)$$

if j<0, we define $c_j = 0$ for consistency, since $b_0 = 1$, equation (28) becomes a set of M linear equations for the M unknown denominator coefficients.

$$\begin{bmatrix} c_{L-M+1} & c_{L-M+2} & c_{L-M+3} & \cdots & c_L \\ c_{L-M+2} & c_{L-M+3} & c_{L-M+4} & \cdots & c_{L+1} \\ c_{L-M+3} & c_{L-M+4} & c_{L-M+5} & \cdots & c_{L+2} \\ \vdots & \vdots & \vdots & & \vdots \\ c_L & c_{L+1} & c_{L+2} & & c_{L+M-1} \end{bmatrix} \begin{bmatrix} b_M \\ b_{M-1} \\ b_{M-2} \\ \vdots \\ b_1 \end{bmatrix} = \begin{bmatrix} c_{L+1} \\ c_{L+2} \\ c_{L+3} \\ \vdots \\ c_{L+M} \end{bmatrix} \qquad (31)$$

from these equations b_i may be found. The numerator coefficients $a_0,.a_1,...,a_L$, follow immediately from equation (29) by equating the coefficient of $1,\eta,\eta^2,...,\eta^{L+M}$

$$\begin{aligned} a_0 &= c_0, \\ a_1 &= c_1 + b_1 c_0, \\ a_2 &= c_2 + b_1 c_1 + b_2 c_0, \\ &\vdots \\ a_L &= c_L + \sum_{i=1}^{\min(L,M)} b_i\, c_{L-i} \end{aligned} \qquad (32)$$

Thus equation (31) and (32) normally determine the Padé numerator and denominator and are called the Padé equations . The Padé approximant [L, M] is constructed which agrees with $\sum_{i=0}^{\infty} c_i \eta^i$, through order η^{L+M}. For more details reader is referred to (Baker [10]). The [19, 19] Padé approximants of equations (24) & (25) are as follows;

$f(Y)_{[19,19]}$ = $(5.000 - 26.1064\,Y + 70.6315\,Y^2 - 129.653\,Y^3 + 175.511\,Y^4 + 6.75677\times10^7\,Y^5 + 3.61671\times10^8\,Y^6 + 8.70878\times10^8\,Y^7 + 1.25691\times10^9\,Y^8 + 1.19832\times10^9\,Y^9 + 7.6752\times10^8\,Y^{10} + 3.02937\times10^8\,Y^{11} + 3.44733\times10^7\,Y^{12} - 4.37655\times10^7\,Y^{13} - 3.64027\times10^7\,Y^{14} -1.62082\times10^7\,Y^{15} - 4.91371\times10^6\,Y^{16} - 1.06456\times10^6\,Y^{17} – 158943.\,Y^{18} - 13633.6\,Y^{19}) / (1.0000 - 5.42128\,Y + 15.5293\,Y^2 -31.2288\,Y^3 + 49.4131\,Y^4 + 1.35135\times10^7\,Y^5 + 6.96315\times10^7\,Y^6 + 1.64556\times10^8\,Y^7 + 2.34386\times10^8\,Y^8 + 2.20962\times10^8\,Y^9 + 1.40287\times10^8\,Y^{10} + 5.51247\times10^7\,Y^{11} + 6.24312\times10^6\,Y^{12} - 8.0304\times10^6\,Y^{13} - 6.67647\times10^6\,Y^{14} - 2.97331\times10^6\,Y^{15} - 904165\,Y^{16} - 195536\,Y^{17} - 29133\,Y^{18} - 2510.79\,Y^{19})$, (33)

$\theta(\eta)_{[19,19]}$ $=(1.000 - 7.67876\,Y + 26.0853\,Y^2 - 35.4209\,Y^3 - 98.6687\,Y^4 + 5.67155\times10^7\,Y^5 + 3.20385\times10^8\,Y^6 + 5.74111\times10^8\,Y^7 + 6.45632\times10^8\,Y^8 + 2.82153\times10^8\,Y^9 - 4.41758\times10^7\,Y^{10} - 1.57209\times10^8\,Y^{11} -1.07599\times10^8\,Y^{12} - 2.8365\times10^7\,Y^{13} + 1.75597\times10^7\,Y^{14} - 7.24145\times10^6\,Y^{15} + 5.28309\times10^6\,Y^{16} - 1.84955\times10^6\,Y^{17} + 290872\,Y^{18} - 17404.1\,Y^{19}) / (1.000 - 7.28874\,Y + 27.5809\,Y^2 - 72.015\,Y^3 + 145.476\,Y^4 + 5.67145\times10^7\,Y^5 + 3.42509\times10^8\,Y^6 + 9.53736\times10^8\,Y^7 + 1.61142\times10^9\,Y^8 + 1.81136\times10^9\,Y^9 + 1.36695\times10^9\,Y^{10} + 6.13164\times10^8\,Y^{11} + 2.43015\times10^7\,Y^{12} - 2.07572\times10^8\,Y^{13} - 1.90509\times10^8\,Y^{14} -1.02618\times10^8\,Y^{15} - 3.85621\times10^7\,Y^{16} - 1.03811\times10^7\,Y^{17} - 1.91914\times10^6\,Y^{18} – 201120\,Y^{19})$. (34)

VII. RESULTS AND DISCUSSION

In this paper, the DTM-Padé is applied successfully to find analytical solution of discussed problem. Figs 2 to 11 show the results (stream line and temperature profiles) obtained by DTM- Padé method for different values of buoyancy parameter λ, suction parameter Fw, Prandtl number Pr, radiation parameter N, Eckert number Ec and dimensionless stream function (Ψ).

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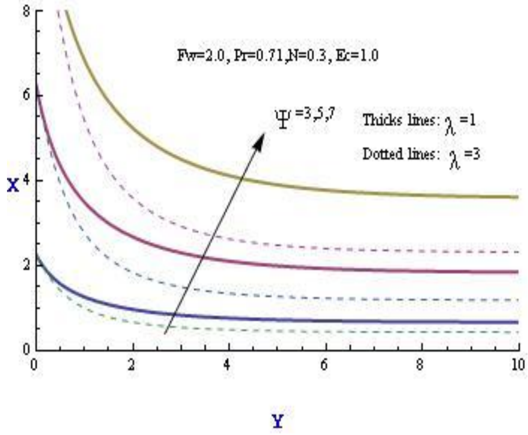

Fig. 2: Streamline profiles for different values of buoyancy parameter (λ) and dimensionless stream function (Ψ)

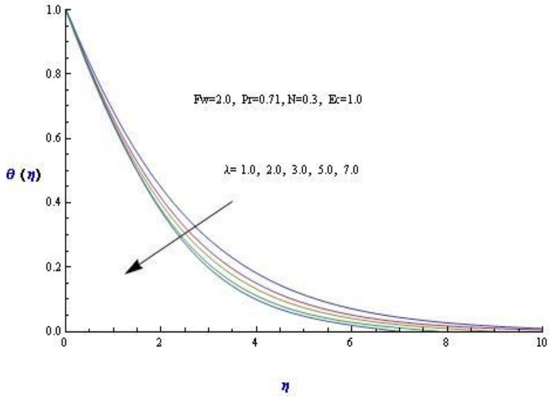

Fig. 3: Temperature profiles for different values of buoyancy parameter (λ)

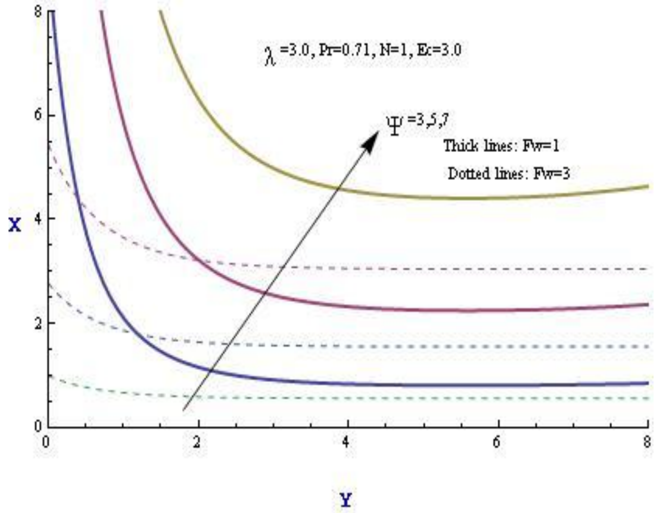

Fig. 4: Streamline profiles for different values of suction parameter (Fw) and dimensionless Stream function (Ψ)

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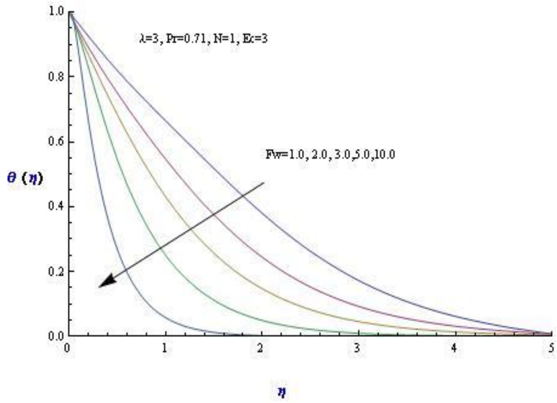

Fig. 5: Temperature profiles for different values suction parameter (Fw)

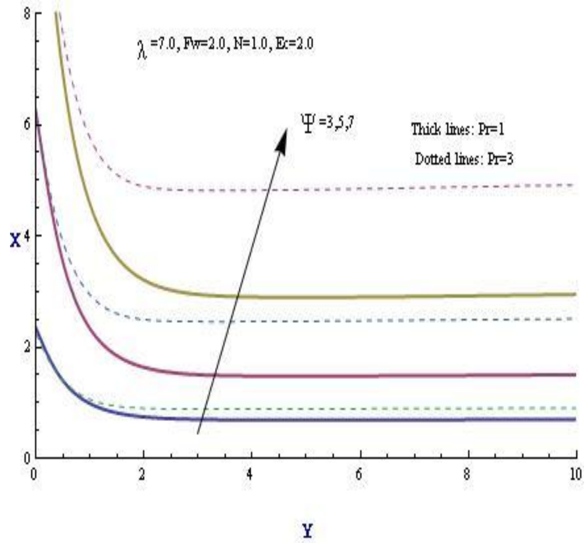

Fig. 6: Streamline profiles for different values of Prandtl number (Pr) and dimensionless stream function (Ψ)

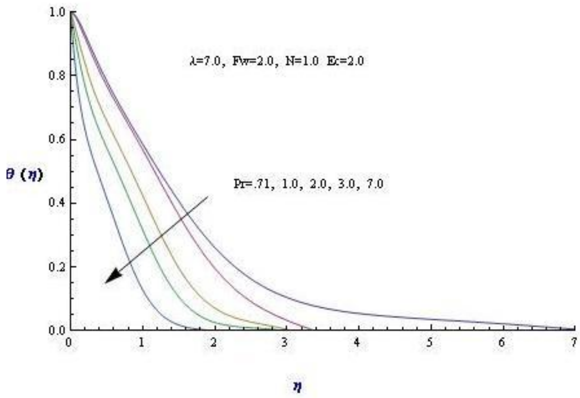

Fig. 7: Temperature profiles for different values Prandtl number (Pr)

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Volume 2, Issue 4, July 2013

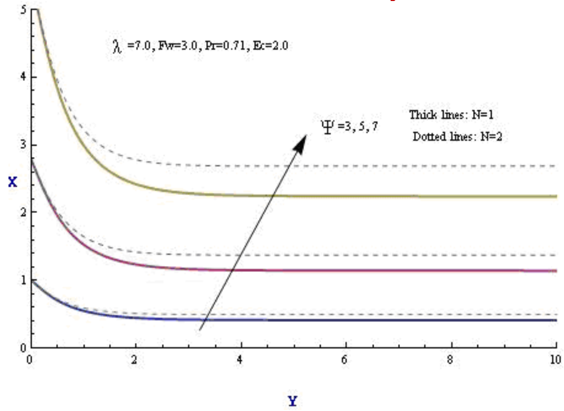

Fig. 8: Streamline profiles for different values of radiation parameter (N) and dimensionless stream function (Ψ)

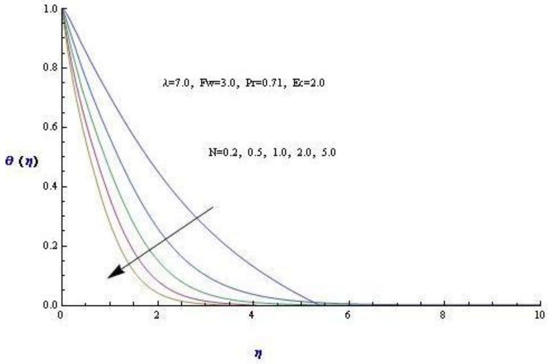

Fig. 9: Temperature profiles for different values radiation parameter (N)

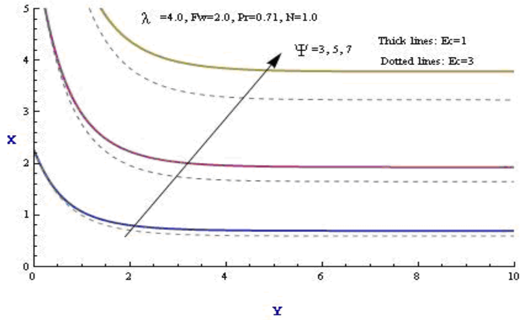

Fig. 10: Streamline profiles for different values of Eckert number (Ec) and dimensionless stream function (Ψ)

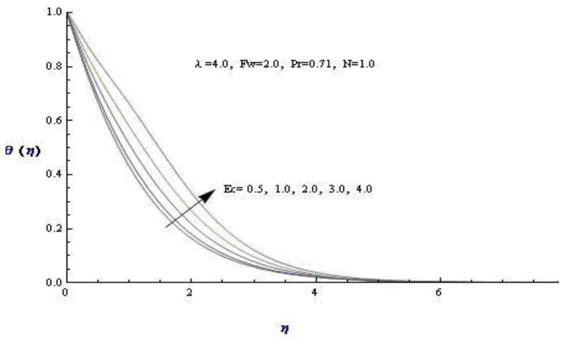

Fig. 11: Temperature profiles for different values Eckert number (Ec)

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VIII. CONCLUSION

In this study the differential transform method combined with the Padé approximant has been used to solve nonlinear differential equations. This method constructs an analytical solution. in the form of a polynomial. we demonstrated that the DTM combined with the Padé approximant provide a reliable tool for solving nonlinear problems in science and engineering .

Nomenclature

c_p: specific heat at constant temperature
D: stretching constant
Dm: coefficient of mass diffusivity
Ec: Eckert number
f : dimensionless stream function
F_w: suction parameter
Gr; Grashof number
k : thermal conductivity
k_1: mean absorption coefficient
N: radiation parameter
q_r: radiation heat flux, Rosseland Approximation
Pr: Prandtl number
Re: Reynolds number
T_w: fluid temperature with in the boundary layer
u: velocity along x-axis
v: velocity along y-axis
v_w: suction velocity
x: coordinate along the plate
y: coordinate normal to the plate

Greek symbols
β : Coefficient of volume expansion
δ : Characteristic length scale
λ : Buoyancy parameter
θ : Dimensionless temperature
ρ : Density of the fluid
σ_1 : Stefan-Boltzmann constant
ψ : Stream function
ν : Kinemat**ic viscosity**

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