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Numerical Modeling of Slope Stability Analysis

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Abstract— ***Limit equilibrium types of analyses for assessing the stability of earth slopes have been in use in geotechnical engineering for many decades. This paper describes numerical simulation study of slope stability problems by using the computer based geotechnical software code Slope/w (Geo-slope 2007). The factor of safety (FOS) has been determined using the limit equilibrium within the Morgenstern–Price method along with Mohr-Coulomb expression. The influence of pore water pressure, cohesion, internal friction angle, and unit weight of upper soil layer on the factor of safety for slope stability problems were investigated through a series of examples. The results show that the factor of safety of the slope stability increases with an increase in cohesion and internal friction angle. However, more pore water pressure present in the soil layer causes a significant reduction in the factor of safety. In addition, the increase in unit weight of soil layer produces lower factor of safety.***

Index Terms— **Slope stability, limit equilibrium, factor of safety, cohesion, internal friction angle, pore water pressure, unit weight.**

I. INTRODUCTION

The slope stability analyses are performed to assess the safe and economic design of human-made or natural slopes (e.g. embankments, road cuts, open-pit mining, excavations, and landfills). In the assessment of slopes, engineers primarily use factor of safety values to determine how close or far slopes are from failure. When this ratio is greater than 1, resistive shear strength is greater than driving shear stress and the slope is considered stable. When this ratio is close to 1, shear strength is nearly equal to shear stress and the slope is close to failure, if FS is less than 1 the slope should have already failed. Limit equilibrium types of analysis for assessing the stability of earth slopes have been in use in geotechnical engineering for many decades. The software code Slope/w(Geo-slope 2007)[1] allows geotechnical engineers to carry out limit equilibrium slope stability analysis of existing natural slopes, unreinforced man-made slopes, or slopes with soil reinforcement. The program uses many methods such as: Bishop's Modified method, Janbu's Simplified method, Spencer method, Morgenstern-Price method and others. Slope/w allows these methods to be applied to circular, composite, and non-circular surfaces.

In 1916, [2] presented the stability analysis of the Stigberg Quay in Gothenberg, Sweden where the slip surface was taken to be circular and the sliding mass was divided into slices. [3] introduced the Ordinary or Swedish method of slices. [4],[5] developed advances in the method. The soil mass above a trial failure surface is divided into slices by vertical planes. Each slice is taken as having a straight line base. The Factor of Safety of each slice is assumed to be the same, implying mutual support between the slices, i.e. there must be forces acting between the slices. Figure 1 shows a typical sliding mass discredited into slices and the possible forces on the slice.

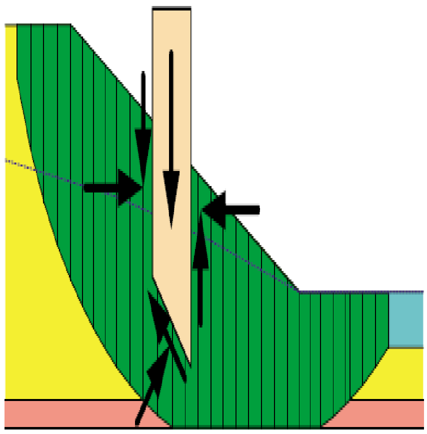

Fig 1 Slice discretization and slice forces in a sliding mass [1]

Normal and shear forces act on the slice base and on the slice sides as shown in figure 2.

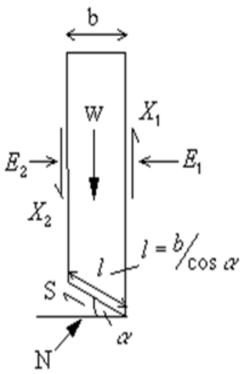

Fig 2 Forces acting on each slice [1].

E1, E2 shear forces acting between adjacent elements.
X1, X2 total normal forces on the sides of the element.
W - Weight of the slice; b- width of the slice.
N- Total normal force on the base of the slice; S- shear force mobilized on the base of the slice.
α - Angle between the tangent to the center of the base of each slice and the horizontal
All limit equilibrium methods utilize the Mohr-Coulomb expression to determine the shear strength □f along the sliding surface.

II. LIMIT EQUILIBRIUM METHODS

A. General Limit Equilibrium Method

A general limit equilibrium (GLE) formulation was developed by Fredlund at the University of Saskatechewan [6],[7]. This method encompasses the key elements of all the other methods available in Slope/W. The GLE formulation is based on two factors of safety equations. One equation gives the factor of safety with respect to moment equilibrium, F_m (equation 1) while the other equation gives the factor of safety with respect to horizontal force equilibrium, F_l (equation 2).

$$F_m = \frac{\sum (c' \beta R + (N - u \beta) R \tan(\phi'))}{\sum W x - \sum N f \pm \sum D d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$F_l = \frac{\sum (c' \beta \cos(\alpha) + (N - u \beta) \tan(\phi') \cos(\alpha))}{\sum N \sin(\alpha) - \sum D\cos(\omega)} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

The terms in the equations are:
c' - effective cohesion ; ϕ' – effective angle of friction ; β, R, x, f, d, ω - geometric parameters
u – pore water pressure ; N – slice base normal force ; W – slice weight
D – Concentrated point load; α – inclination of slice base.
F is F_m when N is substituted into the moment factor of safety equation and F is F_l when N is substituted into the factor of safety equation. The GLE method in Slope/W can accommodate a wide range of different interslice force functions such as: constant, half-sine, clipped-sine, trapezoidal, and data point fully specified. A rigorous review of equilibrium method of slope stability can be found in [8].

B. Ordinary of Fellenius method

The simplest form of the Ordinary factor of safety equation in the absence of any pore-pressures for a circular slip surface is given in the equation (3):

$$F.S = \frac{\sum (c \ \beta + N \tan(\phi')}{\sum W \sin(\alpha)} = \frac{\sum S_{Resistance}}{\sum S_{Mobilized}} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Where, c is the cohesion, β is the slice base length, N is the base normal (W cos(α)), ϕ is the friction angle, W is the slice weight, and α is the slice base inclination.

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C. Bishop's simplified method

In 1950's simplified methods of [4] which included inter-slice normal forces, but ignored the inter-slice shear forces. A simple form of the Bishop's simplified factor of safety equation in the absence of any pore water pressure is:

$$F.S = \frac{1}{\sum W \sin(\alpha)} \sum \left[\frac{c\,\beta + W\tan(\phi) - \frac{c\,\beta}{FS}\sin(\alpha)\tan(\phi)}{m_a} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

FS is on both sides of the equation as noted above. The equation is not unlike the Ordinary factor of safety equation except for the m_a term, which is defined as:

$$m_a = \cos(\alpha) + \frac{\sin(\alpha)\tan(\phi)}{FS} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

D. Morgenstern-Price Method.

This method was developed by [9],[10], which consider not only the normal and tangential equilibrium but also the moment equilibrium for each slice in circular and non-circular slip surfaces. It is solved for the factor of safety using the summation of forces tangential and normal to the base of a slice and the summation of moments about the center of the base of each slice. The equations were written for a slice of infinitesimal thickness. The force and moment equilibrium equations were combined and a modified Newton-Raphson numerical technique was used to solve for the factor of safety satisfying force and moment equilibrium. The solution required an arbitrary assumption regarding the direction of the resultant of the interslice shear and normal forces.

III. ANALYSIS OF SLOPE STABILITY BY USING SLOPE/W SOFTWARE

SLOPE/W is the leading software product for computing the factor of safety of earth and rock slopes. With SLOPE/W, both simple and complex problems can be analyzed for a variety of slip surface shapes, pore-water pressure conditions, soil properties, analysis methods and loading conditions. Using limit equilibrium, SLOPE/W can model heterogeneous soil types, complex stratigraphic and slip surface geometry, and variable pore-water pressure conditions using a large selection of soil models. SLOPE/W gives essentially the same factor of safety as the published solutions by [7], who used the stability programs from University of Alberta and the University of Saskatchewan. This confirms that SLOPE/W is formulated correctly.

A. Factor of safety determination for different cases of pore water pressure

In the assessment of slopes, engineers primarily use factor of safety values to determine how close or far slopes are from failure. The factor of safety is the same for all slices. Pore water pressure is an important factor to be considered in slope stability analysis.

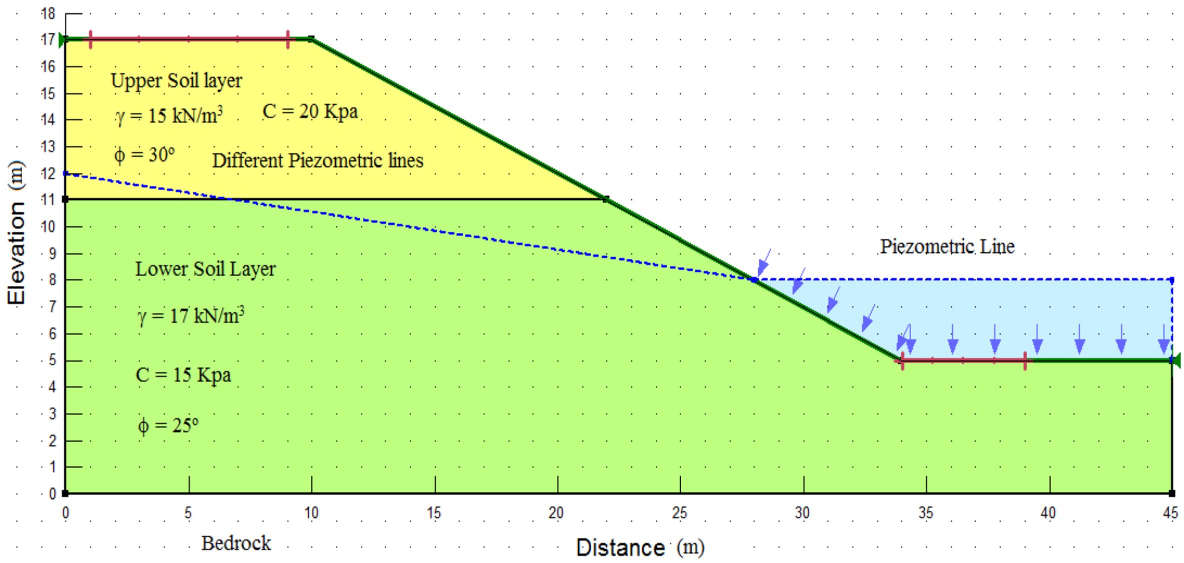

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Figure 3 Profile and material property information for two-dimensional slope stability problem, For different piezometric lines. To keep the analysis simple, only four different levels of pore water pressure (piezometric line) were used in this example: 1m, 3 m, 5 m, 8 m. The results of the example are summarized in Table 1. It is worth noticing that the factors of safety ranged from 1.522 to 1.811. This shows that more pore water pressure present in the layer causes a significant reduction in Factor of safety.

Example 1: Figure 3 shows a schematic diagram of two-dimensional slope stability problem, the geometry and material properties are presented for each layer. For the upper soil layer, the unit weight $\gamma = 15$ kN/m^3, cohesion c = 20 kN/m^2, friction angle $\phi = 30^o$. However, for the lower soil layer, the unit weight $\gamma = 17$ kN/m^3, cohesion c = 15 kN/m^2, friction angle $\phi = 25^o$. The pore water pressure is described by the depicted piezometric line. The purpose of this example is to compute the factor of safety for different pore water pressure level (piezometric line). The slope is cut in two materials at 2:1 (horizontal: vertical). The upper layer is 6 m thick and the total height of the cut is 12 m. Bedrock exists 5 m below the base of the cut.

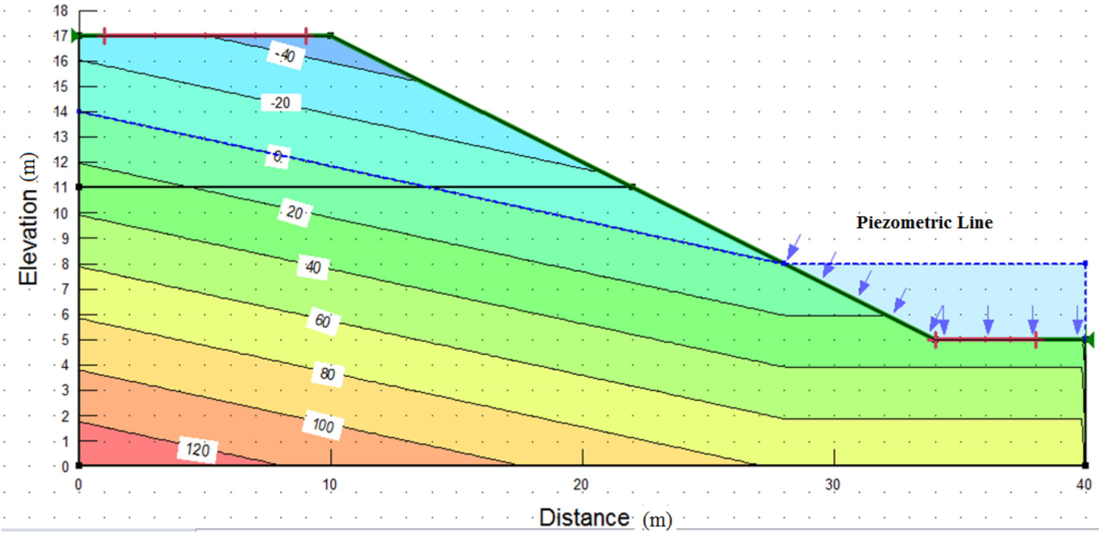

Fig 4 Pore pressure from a piezometric line

Free body diagram and force polygon for the slice 15 is shown in figure 5. The force polygon close indicating the slice is in force equilibrium.

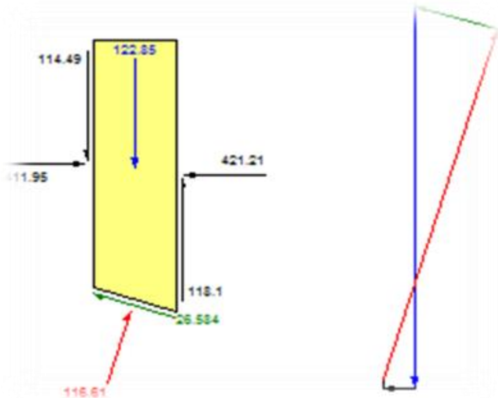

Fig 5 Free body diagram and force polygon closure for Slice 15 of the slip surface

Table 1: Comparison of factor of safety values with different depth of piezometric line below the upper soil layer surface

Case	Upper soil layer			Upper soil layer			Depth of Piezometric Line below the upper soil layer surface	Factor of safety (FOS)
	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})		
1	20	15	30	15	17	25	1 m	1.522

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2	20	15	30	15	17	25	3 m	1.624
3	20	15	30	15	17	25	5 m	1.708
4	20	15	30	15	17	25	8 m	1.811

The variation of the depth of piezometric line below the upper soil layer surface and factor of safety is shown in figure 6. The factor of safety after the lowering of the ground water table (pore water pressure) increased from 1.522 to 1.624 and from 1.708 to 1.811 for lowering the ground table from 1m to 3 m and from 5 m to 8 m respectively. It is worth noticing that a linear relationship exists between the two parameters, it is observed that when lowering the pore water pressure from the upper soil layer, the shear strength of the soil increases, therefore, the factor of safety for slope failure increases. As the pore water pressure can alter the cohesion and frictional parameters through the pores of soil particles which cause a decrease in the strength of the soil.

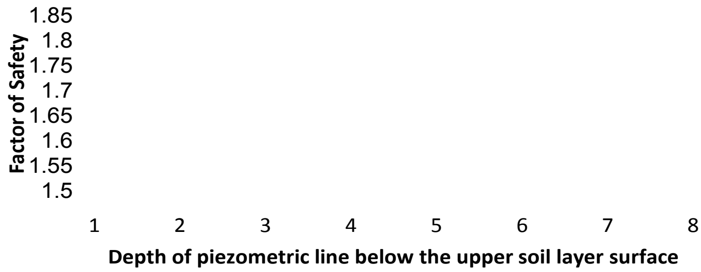

Fig 6 Relationship between depth of piezometric line below the upper soil layer surface and factor of safety from example 1

Figure 7 shows the factor of safety and slip surface results of (Morgenstern and Price, 1965) analysis computed by SLOPE/W for example 1 at 3 m depth of piezometric line below the upper soil layer surface. Figure 8 shows the Safety map of the location of the critical slip surface with respect to all trial slip surfaces.

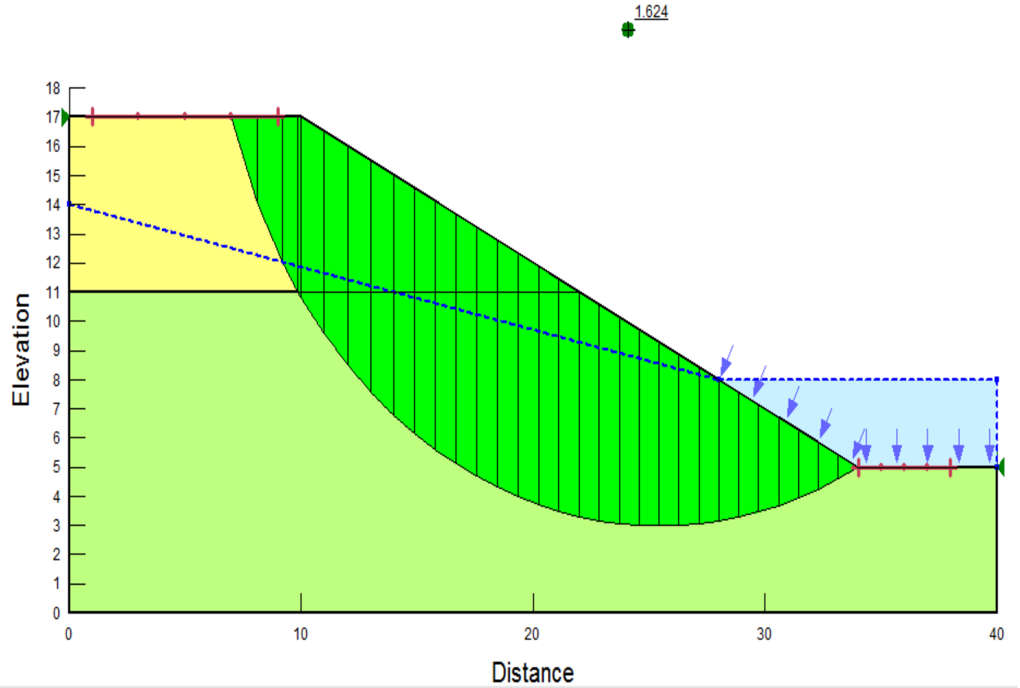

Fig 7 Factor of safety and slip surface results of Morgenstern-Price analysis computed by SLOPE/W

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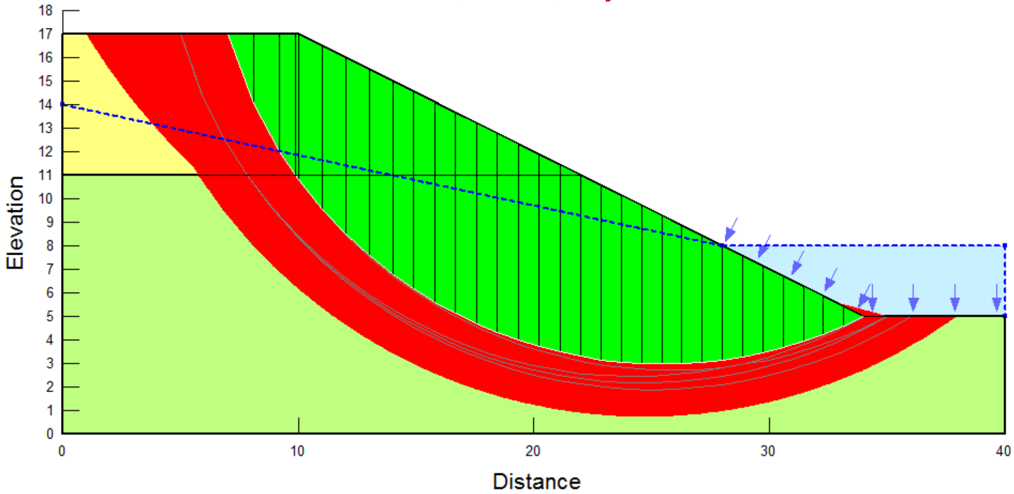

Fig 8 Safety map of the location of the critical slip surface with respect to all trial slip surfaces

B. Influence of Unit Weight of the upper soil layer

Example 2: Figure 9 shows a schematic diagram of two-dimensional slope stability problem, the geometry and material properties are presented for each layer. For the upper soil layer, the unit weight $\gamma = 15$ kN/m^3, cohesion c = 20 kN/m^2 , friction angle $\phi = 30^o$. However, for the lower soil layer, the unit weight $\gamma = 17$ kN/m^3, cohesion c = 15 kN/m^2 , friction angle $\phi = 25^o$. The pore water pressure is described by the depicted piezometric line. The purpose of this example is to compute the factor of safety for different unit weight of the upper soil layer. The slope is cut in two materials at 2:1 (horizontal: vertical). The upper layer is 6 m thick and the total height of the cut is 12 m. Bedrock exists 5 m below the base of the cut.

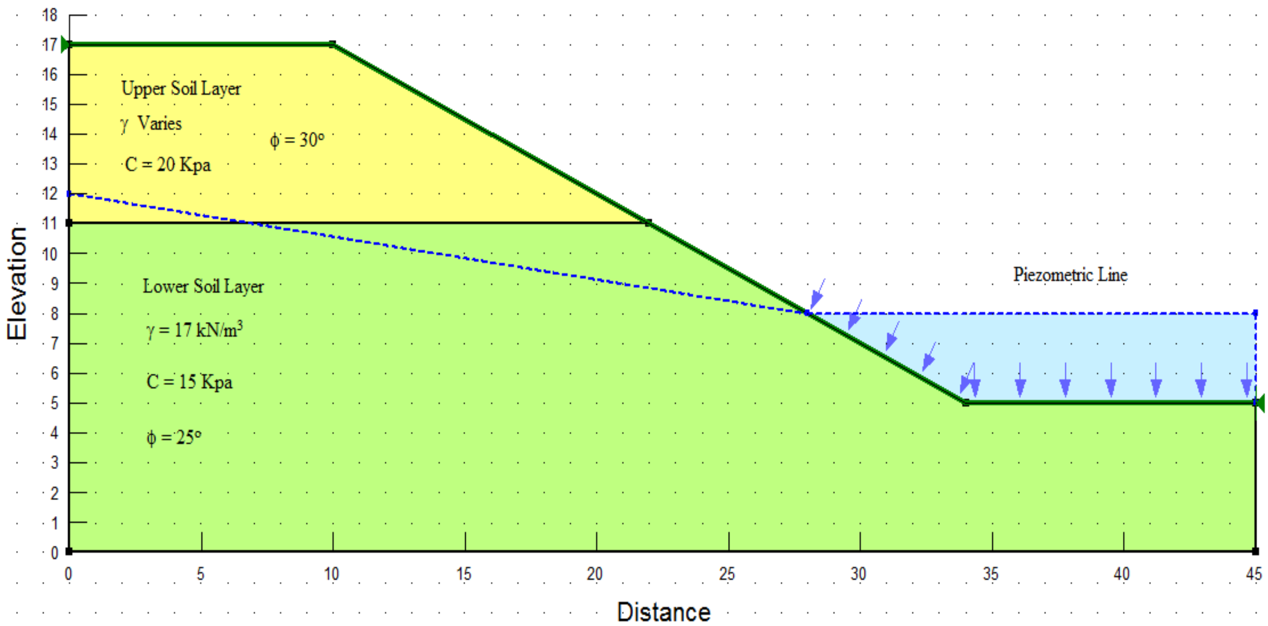

Fig 9 Profile and material property information for two-dimensional slope stability problem For different unit weight values

In order to investigate the influence of the unit weight of the upper soil layer in slope stability analysis, four different values of unit weight are considered: 12 kN/m^3, 14 kN/m^3, 16 1 kN/m^3, and 18 kN/m^3 . The results of the example are summarized in Tables 2. Figure 10 shows the variation of factor of safety for the increase in unit weight of upper soil layer. The value of factor of safety decreases with the increase in unit weight of upper soil layer, the variation ranged from 1.601 to 1.840 as shown in figure 10. It is observed that with an increase in unit weight there is a decrease in the factor of safety. A linear relationship exists between the two parameters.

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Table 2: Factor of safety results for different values of unit weight

Case	Upper soil layer			Upper soil layer			Factor of safety (FOS)
	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})	
1	20	12	30	15	17	25	1.840
2	20	14	30	15	17	25	1.748
3	20	16	30	15	17	25	1.669
4	20	18	30	15	17	25	1.601

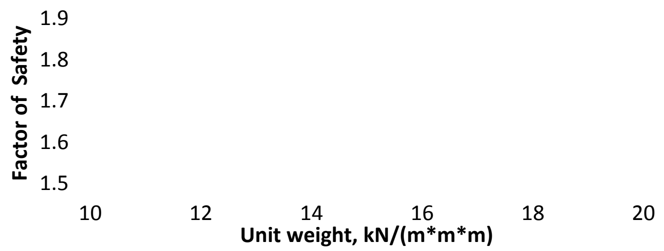

Fig 10 Relationship between unit weight and factor of safety of upper soil layer slope

C. Influence of cohesion of the upper soil layer

Example 3: Figure 11 shows a schematic diagram of two-dimensional slope stability problem, the geometry and material properties are presented for each layer similar to example 2.

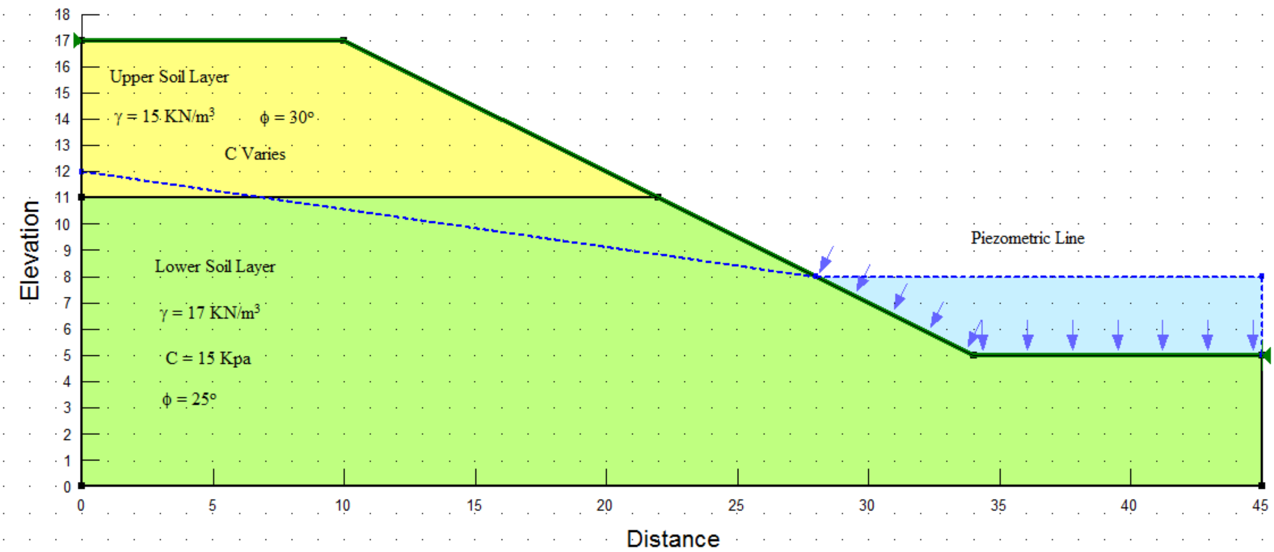

Fig 11. Profile and material property information for two-dimensional slope stability problem for different cohesion

The purpose of this example is to compute the factor of safety for different values of cohesion. The slope is cut in two materials at 2:1 (horizontal: vertical). The upper layer is 6 m thick and the total height of the cut is 12 m. Bedrock exists 5 m below the base of the cut. Five different cohesions (0 kPa, 10 kPa, 20 kPa, 30 kpa, 40 kPa) are adopted to obtain the slope stability and critical failure mechanism, the achieved factor of safety and the solutions from Slope/W. The factors of safety ranged from 1.560 to 1.817 as listed in Table 3.

Table 3: Factor of safety results for different values of cohesion

Case	Upper soil layer			Upper soil layer			Factor of safety (FOS)
	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})	
1	0	15	30	15	17	25	1.560

2	10	15	30	15	17	25	1.645
3	20	15	30	15	17	25	1.707
4	30	15	30	15	17	25	1.765
5	40	15	30	15	17	25	1.817

The effect of cohesion on the stability of the upper soil layer slope is shown in Figure 12. Figure 12 is a plot of relationship between cohesion and factor of safety of upper soil layer slope from example 3. It is observed that the factor of safety increases linearly with an increase in cohesion. This is probably due to linear relationship of cohesion with shear strength as defined by the Mohr-Coulomb failure criterion.

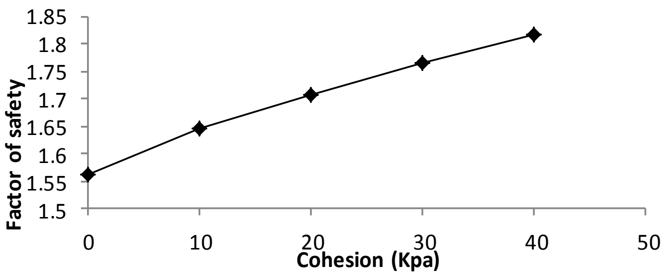

Fig12 Relationship between cohesion and factor of safety of upper soil layer slope

D. Influence of internal friction angle of the upper soil layer

Example 4: Figure 13 shows a schematic diagram of two-dimensional slope stability problem, the geometry and material properties are presented for each layer similar to example 3.
The purpose of this example is to compute the factor of safety for different internal friction angles. The slope is cut in two materials at 2:1 (horizontal: vertical). The upper layer is 6 m thick and the total height of the cut is 12 m. Bedrock exists 5 m below the base of the cut.

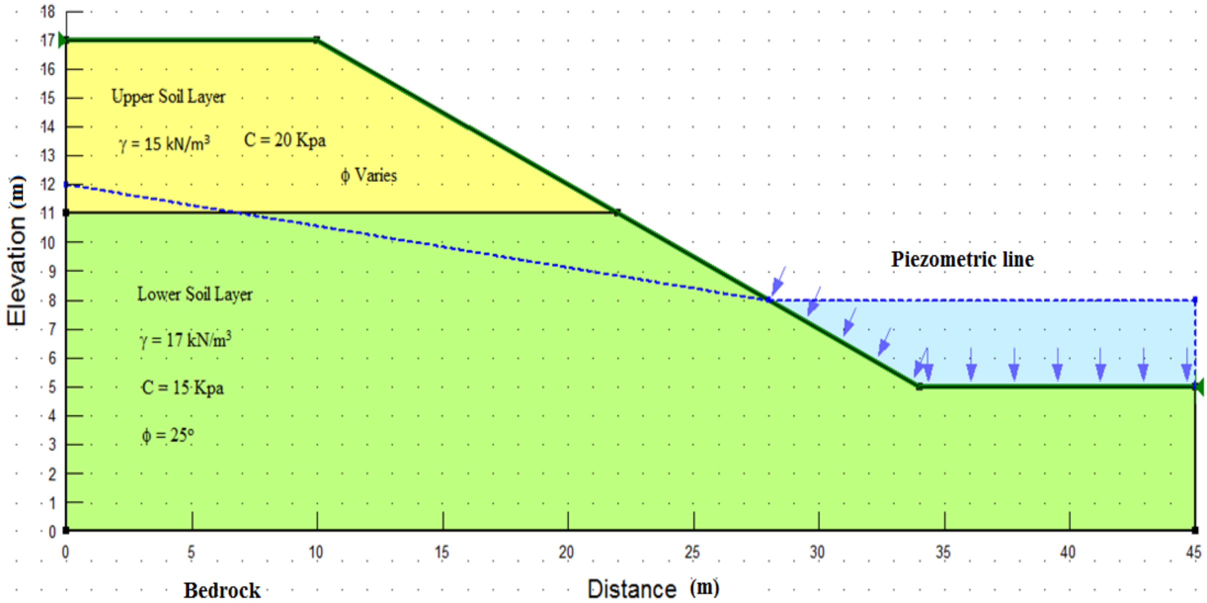

Fig 13 Profile and material property information for two-dimensional slope stability problem for friction angle values

Five different internal friction angles (0°, 10°, 20°, 30° , and 40°) are adopted for the upper soil layer to obtain the slope stability and critical failure mechanism, the achieved factor of safety and the solutions from Slope/W are listed in table 4. The factors of safety ranged from 1.588 to 1.727. The relationship between the internal friction angle and factor of safety of upper soil layer slope is shown in Figure 14. These results show that when the internal friction angle ϕ increases, the safety factor of the slope increases.

Table 4: Factor of safety results for different values of friction angle

Case	Upper soil layer			Upper soil layer			Factor of safety (FOS)
	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})	Cohesion (Kpa)	Unit Weight (kN/m^3)	Friction angle (^{o})	
1	20	15	0	15	17	25	1588
2	20	15	10	15	17	25	1.637
3	20	15	20	15	17	25	1.685
4	20	15	30	15	17	25	1.707
5	20	15	40	15	17	25	1.727

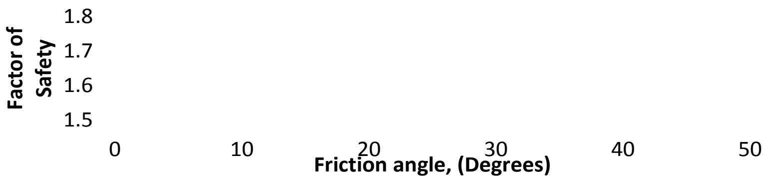

Fig 14 Relationship between internal friction angle and factor of safety of upper soil layer slope

It is observed that with an increase in friction angle, Factor of Safety increases almost linearly for the first three values of friction angles of 0^o, 10^o, and 20^o and then the line takes another straight line for the remaining values of friction angles of 30^o, and 40^o. It is observed in general that the factor of safety increases almost linearly with an increase in ϕ. This is also probably due to linear characteristics of the Mohr-Coulomb failure criterion used in the present study and as the internal friction angle related between normal force and resultant force which withstand the shear stress.

IV. CONCLUSION

The following conclusions have been drawn from this study :

- The study of slope stability problems by using the computer based geotechnical software code Slope/w provides more understanding viewing all the detailed forces on each slice, to understand failure mechanisms, and the distribution of a variety of parameters along the slip surface with respect to the factor of safety.
- As the cohesion is the characteristic property of soil that measures how well it resists being deformed or broken by different forces on slope stability problem. It was found that the factor of safety increases linearly with increasing the cohesion of the soil of slope stability for values greater than 10 kPa. Analysis of the effect of internal friction angle on slope stability showed that the factor of safety increases nonlinearly with increasing the internal friction angle.
- The factor of safety of the slope decreases almost linearly with an increase in unit weight and decreases nonlinearly with lowering the pore water pressure from the ground surface level of the upper soil layer of the slope stability.
- From the results of numerical calculations, it is found that the four parameters studied have significant influence on the safety factor of slope stability problem.

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