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Identification of Time Varying System Using Recursive Estimation Approach and Wavelet Based Recursive Estimation Approach

Sandesh N. Mate[1], Ajay B. Patil[2]
[1,2]Walchand College of Engineering, Sangli, India

***Abstract*— Identification problem of time varying system can be solved Recursive estimation method and Wavelet based recursive estimation method. There are many processes which contain nonstationary events; identification of such system can be achieved by parameter adaption algorithms. In this paper the identification problem is solved by normalized LMS algorithm and Wavelet based estimation method. Performance of these two algorithms can be judged by comparing results.**

***Index Terms*— Time varying systems, parameter estimation, system identification, LMS algorithm, Multi-Wavelet basis function.**

I. INTRODUCTION

There are many systems which fail to satisfy the stationary assumptions. These systems contain some on-stationary events and are explicitly dependent on time. These type of systems are known as time varying systems. We cannot use time invariant models to characterize these type of systems. We require time-varying parametric models like Time-varying autoregressive with an exogenous (TVARX) or Time varying autoregressive (TVAR) models for the modeling and analysis of time varying systems. Method of parameter estimation is known as Adaptive recursive estimation.Adaptive recursive estimation method is called as stochastic approach, where coefficients of the associated model are treated as random processes with some stochastic model structure. Adaptive algorithms used in this approach are LMS algorithm, RLS algorithm and Kalman filter algorithm.

System identification is an experimental approach for determining the dynamic model of a system. An advantage of system identification is that it takes a lot less time than physical modeling since it is only concerned with input and output signals from the system. There are different approaches for model identification and parameter estimation of time varying systems. Assume a nonstationary process is locally stationary and Least Square (LS) techniques are applied. This method gives good results for stationary process but not suitable for rapidly time varying systems. The second method, Basic function expansion and regression method is called as deterministic regression approach, where associated time varying coefficients are expanded using linear or nonlinear combination of finite number of basis functions. The problem then becomes time invariant with respect to the parameters in the expansion and can be solved by regression selection and parameter estimation. [1]

In this paper we employed the Time Varying Auto-Regressive with exogenous (TVARX) model and its parameters are estimated using recursive methods. For experimental simulation, we have used the first order TVARX model and its parameters are estimated. The input is chosen to Pseudo-Random Binary Sequence (PRBS) which is frequency rich signal. The results are taken for abruptly varying parameters. The performance is evaluated by comparing true and estimated values.

An attractive approach is to select -wavelet as the basis function. Wavelet can be easily used for approximation general non-stationary signals, even those with sharp discontinuities. Wavelets have been successfully applied to EEG signal processing and analysis, nonlinear system identification and also in many other fields. In proposed approach, multi-wavelet expansion based method is applied to time varying system for time dependent parameter estimation. Decomposition of time dependent parameters involving multi-wavelet expansion scheme can effectively track both smooth as well as non-smooth variation of time varying coefficients, as compared with decomposition involving only single type of wavelet [2].In this approach, time varying parameters of time varying system are approximated by using multi-wavelet basis functions, which converts time varying identification problem into time invariant parametric expansion. The identification of model parameters can then be achieved by using any recursive algorithm. This approach can be used to track smooth as well as non-smooth or sharp variations in process. Hence this approach can be used for identification problem of inherently non-stationary processes. A

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multi-wavelet basis function approach is used because it can capture signal characteristics at different scales. Example for synthetic data set is given to illustrate effectiveness of proposed method [2].

II. LEAST MEAN SQUARE ALGORITHM

The Least Mean Square (LMS) algorithm, introduced by Widrow and Hoff in 1959 is an adaptive algorithm, which uses a gradient-based method of steepest decent. LMS algorithm uses the estimates of the gradient vector from the available data. LMS incorporates an iterative procedure that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. Compared to other algorithms LMS algorithm is relatively simple; it does not require correlation function calculation nor does it require matrix inversions. The least mean squares (LMS) algorithms adjust the filter coefficients to minimize the cost function. Compared to recursive least squares (RLS) algorithms, the LMS algorithms do not involve any matrix operations. Therefore, the LMS algorithms require fewer computational resources and memory than the RLS algorithms. The implementation of the LMS algorithms also is less complicated than the RLS algorithms. However, the eigenvalue spread of the input correlation matrix, or the correlation matrix of the input signal, might affect the convergence speed of the resulting adaptive filter The standard LMS algorithm performs the following operations to update the coefficients of an adaptive filter:

- Calculates the output signal y(n) from the adaptive filter.
- Calculates the error signal e(n) by using the following equation:

e(n) = d(n)–y(n)

- Updates the filter coefficients by using the following equation:

$$w(n+1) = w(n) + \mu\, u(n)e(n)$$

Where, μ is the step size of the adaptive filter, w(n) is the filter coefficients vector, and u(n) is the filter input vector.

Block shematic of LMS algorith is shown below,

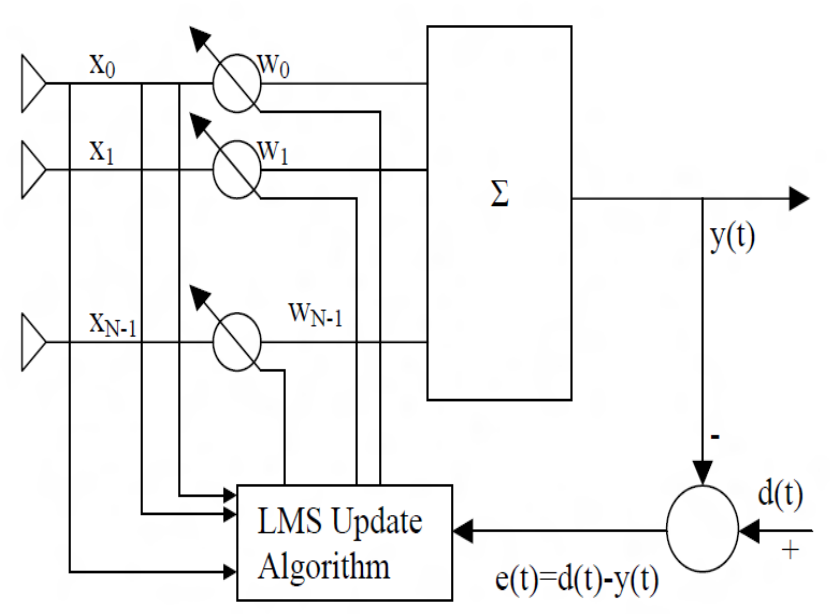

Fig 1. Block schematic of LMS algorithm

A. Normalized LMS algorithm

The normalized LMS (NLMS) algorithm is a modified form of the standard LMS algorithm. The NLMS algorithm updates the coefficients of an adaptive filter by using the following equation:

$$w(n+1) = w(n) + \eta e(n) \frac{u(n)}{\|u(n)\|^2}$$

You also can rewrite the above equation to the following equation:

$$w(n+1) = w(n) + \mu(n)\, x(n)e(n) \qquad (1)$$

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where, $\mu(n) = \frac{\mu}{\|u(n)\|^2}$. In the previous equation, the NLMS algorithm becomes the same as the standard LMS algorithm except that the NLMS algorithm has a time-varying step size $\mu(n)$. This step size can improve the convergence speed of the adaptive filter.

III. MULTI-WAVELET BASIS FUNCTIONS

As we know, any signal can be arbitrarily approximated as,

$$f(x) = \sum_k \alpha_{j0,k}\phi_{j0,k}(x) + \sum_{j=j0}^{\infty}\sum_k \beta_{j,k}\Psi_{j,k}(x) \qquad (2)$$

Where, $\phi_{j0,k}(x) = 2^{\frac{j0}{2}}\phi(2^{j0}x - k)$ and $\Psi_{j,k}(x) = 2^{\frac{j}{2}}\Psi(2^j x - k)$ are the scaled and translated version of the scaling function ϕ(x) and the mother wavelet $\Psi(x)$, $\alpha_{j0,k}$ and $\beta_{j,k}$ are the wavelet decomposition coefficients at the scale level j0 and j respectively. The first summation indicates a low resolution or coarse approximation of f(x) at the scale level j0. For each j, a finer or higher resolution function including more detail of f(x) is added in the second summation. Like in any regression representation of functions, $\phi_{j0,k}(x)$ and $\Psi_{j,k}(x)$ can be interpreted as regressions.

It has been found that B-spline mother wavelets give good results for high resolution wavelet based approximation B-spline as piece-wise polynomial function with good local properties is used as wavelet and scaling functions in multi-resolution expansion. The first order B-spline is well-known Haar function defined as,

$$B_1(x) = X[0,1) = \begin{cases} 1, & x < [0,1) \\ 0, & \text{otherwise} \end{cases}$$

The mth order B-spline is defined on [0,m], with the scale and shift indices j and k, for the family of the function

$$\phi_{j,k}(x) = 2^{\frac{j}{2}}B_m(2^j x - k), \qquad 0 \le 2^j x - k < m \qquad (3)$$

First and second order B-splines $B_1(x)$ and $B_2(x)$ are non-smooth piecewise functions, which would perform well for signals with sharp transients and spikes, B-splines of higher order would work well on smoothly changing signals.

IV. METHODOLOGY FOR WAVELET BASED ESTIMATION APPROACH

Consider TVARX Model which is described by the following equation:

$$y(t) = \sum_{i=1}^{P} a_i(t)\, y(t-i) + \sum_{j=1}^{Q} b_j(t)\, u(t-j) + e(t) \qquad (4)$$

Where, u and y are the sampled measurable input, output, and prediction error signals $a_i(t)$, and $b_j(t)$ are the time-varying parameters to be determined, P and Q are the maximum model orders, and represents discrete time. Time varying coefficients $a_i(t)$ and $b_j(t)$ can be approximated using multi-wavelet basis functions as,

$$a_i(t) = \sum_k \alpha_{j0,k}^{(a,i)}\phi_{j0,k}(t) + \sum_{j=j0}^{J1}\sum_k \beta_{j,k}^{(a,i)}\Psi_{j,k}(t)$$

$$b_i(t) = \sum_k \alpha_{j,k}^{(b,i)}\phi_{j,k}(t) + \sum_{j=j0}^{J2}\sum_k \beta_{j,k}^{(b,i)}\Psi_{j,k}(t)$$

Substituting these in equation (4) we get;

$$y(t) = \sum_{i=1}^{P}\sum_k \alpha_{j0,k}^{(a,i)}\phi_{j0,k}(t)\, y(t-i) + \sum_{i=1}^{P}\sum_{j=j0}^{J1}\sum_k \beta_{j,k}^{(a,i)}\Psi_{j,k}(t)\, y(t-i) + \sum_{j=1}^{Q}\sum_k \alpha_{j0,k}^{(b,i)}\phi_{j0,k}(t)u(t-j) + \sum_{j=1}^{Q}\sum_{j=j0}^{J2}\sum_k \beta_{j,k}^{(b,i)}\Psi_{j,k}(t)\, u(t-j) + e(t)$$

(5)

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Let us define some terms

P(t) = [y(t-1), y(t-2), y(t-3),…….., y(t-P)]
Q(t) = [u(t-1), u(t-2), u(t-3),………, u(t-Q)]
R(t) = [$\phi_{j0,k_0}(t), \phi_{j0,k_0+1}(t), \phi_{j0,k_0+2}(t), \ldots, \phi_{j0,k_{j0}}(t)$]
A(t) = P(t) $\otimes$ R(t)
S(t) = [$\Psi_{j,k_0}(t), \Psi_{j,k_0+1}(t), \Psi_{j,k_0+2}(t), \ldots, \Psi_{j,k_{j0}}(t)$]
B_j(t) = P(t) $\otimes$ S(t)
C(t) = Q(t) $\otimes$ R(t)
D_j(t) = Q(t) $\otimes$ S(t)
B_j(t) = [$B_{j_0}^T$(t), $B_{j_0+1}^T$(t), $B_{j_0+2}^T$(t),…….., $B_{j_1}^T$(t)]
D_j(t) = [$D_{j_0}^T$(t), $D_{j_0+1}^T$(t), $D_{j_0+2}^T$(t),…….., $D_{j_2}^T$(t)]
Here, the symbol “$\otimes$“ denotes the Kronecker product.
By substituting these terms in equation (), we get,
y(t) = A(t) $\alpha^{(a)}$ + B(t) $\beta^{(a)}$ + C(t) $\alpha^{(b)}$ + D(t) $\beta^{(b)}$ +e(t)
If N measurements of the input and output are taken, then above equation can be written in matrix form as,

$$Y = H\theta + \varepsilon \qquad (6)$$

Where,
Y^T = [y(1) y(2) ………y(N)]
ε = [e(1), e(2), e(3),……….e(N)]

$$H = \begin{bmatrix} A(1) & B(1) & C(1) & D(1) \\ A(2) & B(2) & C(2) & D(2) \\ \vdots & \vdots & \vdots & \vdots \\ A(N) & B(N) & C(N) & D(N) \end{bmatrix}$$

$\theta = [\ \alpha^{(a)}\ \beta^{(a)}\ \alpha^{(b)}\ \beta^{(b)}\ ]^T$
Here, θ is the coefficient vector. Equation (5) and (6) shows that TVARX model can now be treated as a time invariant model, since parameters $\alpha_{j0,k}^{(i)}$ and $\beta_{j,k}^{(i)}$ are time invariant.
The parameter θ in equation (6) can now be estimated using LMS based algorithm. However, the number of time dependent time varying parameters in the model is very large, hence Least mean square algorithm may fail to produce correct results. This problem can be solved by applying block LMS algorithm. $\alpha_{j0,k}^{(i)}$ and $\beta_{j,k}^{(i)}$ parameters obtained can be used to recover time varying coefficients $a_i(t)$ and $b_i(t)$ in TVARX model (4).

V. Estimation Of TVARX Model using Recursive Algorithm and Wavelet based method

One method that can be used to resolve the TVARX and TVAR model estimation problem is Recursive estimation of the time-varying coefficients. This approach is called as stochastic approach.. Consider the following TVARX model,

$$y(t) = a_1(t)y(t-1) + b_1(t)u(t-1) + e(t)$$

In this model,$a_1(t)$ and $b_1(t)$are time varying coefficients.
Let’s us assume these parameters as follows,

$$a_1(t) = \begin{cases} -0.1 & 0 \le t \le 0.3 \\ -0.9 & 0.3 < t \le 0 \\ 0.2 & 0.5 < t \le 0.7 \\ -0.5 & 0.7 < t \le 1 \end{cases}$$

$$b_1(t) = \begin{cases} -0.1 & 0 \le t \le 0.2 \\ 0.8 & 0.2 < t \le 0.4 \\ -0.4 & 0.4 < t \le 0.7 \\ 0.6 & 0.7 < t \le 1 \end{cases}$$

Then form a TVARX model using there assumed time varying coefficients. Estimation of this model is then carried out by normalized LMS and Wavelet based recursive estimation method.

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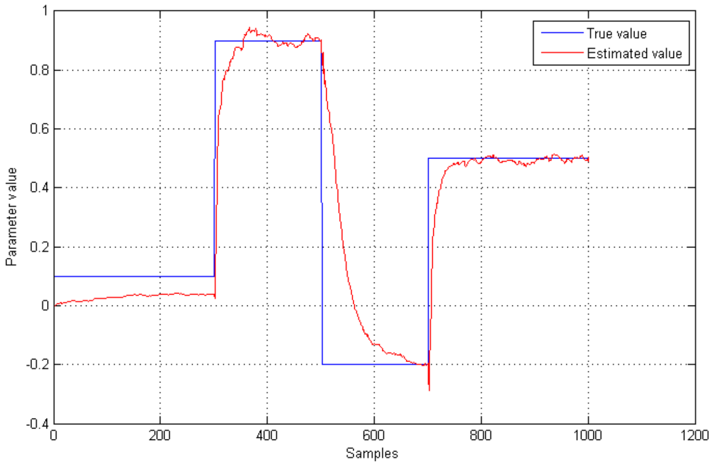
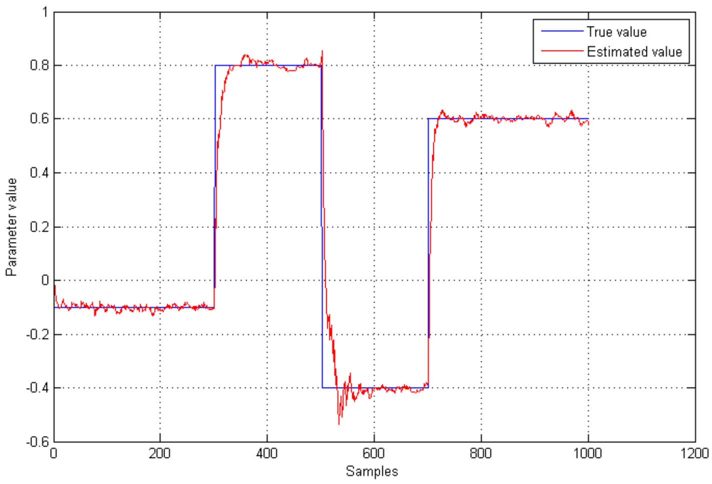
VI. SIMULATION RESULTS

A. Estimation of TVARX model using normalized LMS algorithm.

Estimated and true parameter a(t):



Estimated and true parameters b(t):



B. Estimation of TVARX model using wavelet based approach

In this approach, time varying coefficients are approximated using multi-wavelet basis functions. One such combination is shown below,

$$a_i(t) = \sum_{k\epsilon\Gamma_q} c_{i,k}^{(q)}\phi_k^{(q)}\left(\frac{t}{N}\right) + \sum_{k\epsilon\Gamma_r} c_{i,k}^{(r)}\phi_k^{(r)}\left(\frac{t}{N}\right) + \sum_{k\epsilon\Gamma_s} c_{i,k}^{(s)}\phi_k^{(s)}\left(\frac{t}{N}\right)$$

$$b_j(t) = \sum_{k\epsilon\Gamma_q} d_{j,k}^{(q)}\phi_k^{(q)}\left(\frac{t}{N}\right) + \sum_{k\epsilon\Gamma_r} d_{j,k}^{(r)}\phi_k^{(r)}\left(\frac{t}{N}\right) + \sum_{k\epsilon\Gamma_s} d_{j,k}^{(s)}\phi_k^{(s)}\left(\frac{t}{N}\right) \quad (8)$$

B-splines of order from 1 to 4 are selected for this example.

Here,

$\Gamma_m = \{k: m < k < 2^j\}$, m=1,2,3,4

$\phi_k^{(m)}(x) = 2^{\frac{j}{2}} B_m(2^j x - k)$, $k\,\epsilon\,\Gamma_m$.

$j_0=j=3$

$1 \le q < r < s \le 4$,

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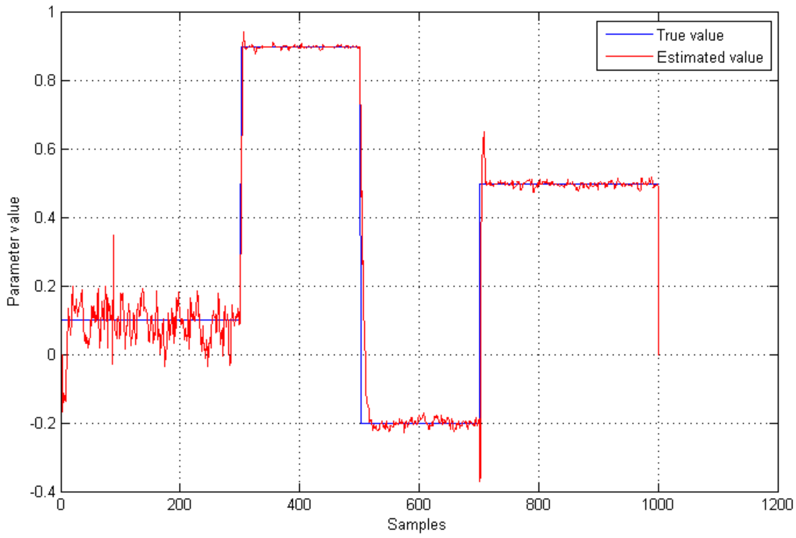
$t = 1,2 \ldots\ldots, N$

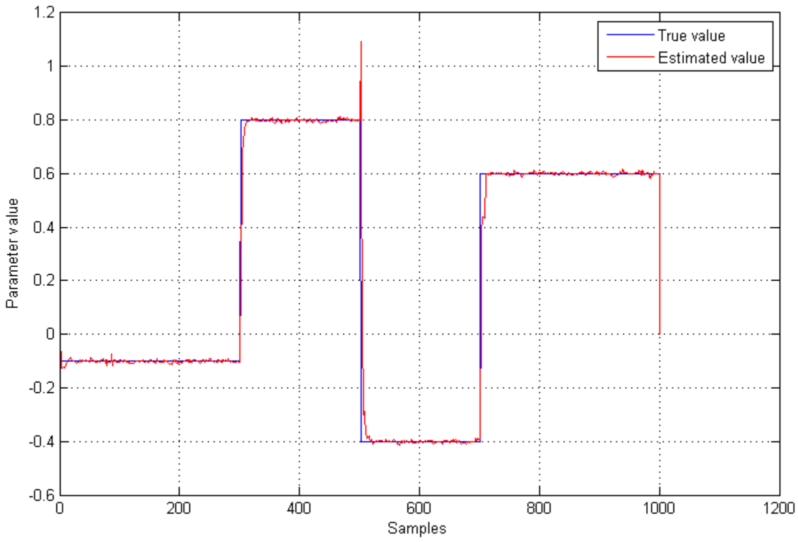
Where, N is number of observations of the signal.

The choice of q=2, r=3 and s=4 work well to recover the time-varying coefficients. the resulting recursive coefficient estimates $c_{i,k}$ and $d_{i,k}$ in (8) will then be used to recover the time-varying coefficients $a_i\ (t)$ and $b_i(t)$ in the TVARX in model (7).

Estimated and true parameter a(t):



Estimated and true parameters b(t):



VII. CONCLUSION

Time-varying parameters in ARX models have been estimated using a new multi-wavelet basis function approach with any recursive algorithm introduced in this study, where the associated time-dependent coefficients are expanded using multi-wavelet basis functions. Parameter variations including abrupt or sharp changes have been considered. Performance measures of the estimated parameters have been calculated. The results indicate that the new approach based on multi-wavelet basis functions with LMS algorithm gives improved results for fast and abrupt changing parameters than the method which uses the traditional normalized LMS algorithm directly. Furthermore, from the results above, it can be concluded that time-varying systems can be modeled using a

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TVARX and the identification problem of modeling fast and abrupt changing time-varying parameters is possible with good accuracy. The identification procedure has been shown to be effective in tracking time evolutions of the unknown parameters. The wavelet method is especially powerful for non-stationary signal analysis.

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AUTHOR BIOGRAPHY

Sandesh N. Mate received the B.E. degree in Electronics and Telecommunication Engineering from KKWIEER college, Nashik from Pune University, Pune in 2010. He is currently pursuing M.Tech degree in Electrical Control System from Walchand College of Engineering, Sangli.

Ajay B. Patil received B.E. degree in Electronics Engineering from K.B.P College of Engineering, Satara, Shivaji University. He received M.E. degree in Control System from Walchand College of Engineering, Sangli. His current research interests include signal processing, fuzzy control, and neural networks.