ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

Shape Optimization of Externally Pressurized Thin-Walled End Dome Closures with Constant Wall Thickness of Cylinder Vessel with Help of Bezier Curve

Jignesh Dipakkumar Jani [1], Manoj Kumar Gupta [2], R. R. Trivedi[3]

Abstract--Reduce stress between junction at knuckle and crown of Torispherical dome because of discontinuity effect, increase smoothness of meridional curvature, and also weight reduction. We use Bezier curve of 5^{th} Degree of Polynomial for Optimization of Dome curvature. Reduce average stress for overall dome and also reduce thickness of material and weight from economics point of view and save material and shape with reduce stress between knuckle and crown. In present study we develop new shape of end dome for Externally Pressurized thin-walled end dome closures and reduce discontinuity effect between knuckle and crown joint and reduce thickness of material and weight of dome to save of material.

Key words: **Knuckle, crown, Discontinuity, Stress, Bèzier curve of 5^{th}Degree of Polynomial, Weight Reduction.**

Nomenclature

- a = internal radius of cylinder
- Lc = length of cylinder
- P = external pressure
- b = total depth of dome
- h = thickness of dome wall
- C1=curvature with help of Bezier curve.
- C2 = Radius of curve no 2, (Standard Curve)
- R = distance from axis of vessel to point of middle surface
- r(z) = shape of middle surface
- R_0=radius of circle and Z_0 = co-ordinate of center of curvature of circle
- bm =part of depth (unknown) for C1 curve
- b_0 = part of depth for curve C2 (circle)
- M = Joining point of C1 and C2 curvature

I. INTRODUCTION

A vessel has many applications in several engineering field like mechanical, chemical, aerospace etc. because they are used to store liquid and gas at high pressure and also use to generate vacuum chamber to several experiments. In general they are use cylindrical shape with end domes (heads). Stress occurs at the junction between cylinder and dome due to change of geometry. Such type of problem can be minimizing, if hemispherical geometry is used for end dome. In certain advance technology like rockets used in space craft, torispherical heads are employed. Two examples of current use are Indian Space Research Organization's (ISRO) Geosynchronous Satellite Launch Vehicle (GSLV) and Space shuttle program of US Space Agency, NASA. The Solid Rocket Boosters (SRBs), liquid and cryogenic stages use the torispherical heads in the tank ages. This paper presents an analysis of stresses in the knuckle and crown region of vessel with torispherical head with help of shape optimization technique like Bezier curve. Due to change of geometry, high stresses are induced in this region and net compressive stresses are in the hoop direction and severe bending stress is in the axial direction. The causes of worry to designers are (i) buckling due to compressive stress (ii) the material failure to high bending stress regions. The torispherical geometry is composed of two circular arcs; (i) the torus region (knuckle) of smaller radius with center of curvature located off the centroid axis and (ii) the spherical dome (crown) with larger radius.

The ratio of the above two radii controls the nature of stress field in the knuckle region and hence is of importance to the designers. [5] Stability behavior of torispherical dome ends under external pressure has not

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013
attracted as much attention as hemispherical or spherical caps over the last few decades, while this kind of domes have been widely used in wide range of industries from air space, land to undersea structures. [1]
Standard shape as per Geometry and ASME torispherical, ellipsoidal or hemispherical head of a vessel significantly disturbs the membrane stress and create discontinuity effect pattern arising in its cylindrical part. The value of the meridional principal curvature of the middle head of end dome surface is non-zero while in the cylindrical it takes zero level. In result the curvature of end dome becomes discontinuous. The present work illustrates an attempt to solve the strength and optimization problems and deals with shaping the middle surface of heads with the use of a rational Bezier curve. The Bezier curve has been developed by Pierre Etienne Bezier for the Renault Company, France. They are probably the most frequently used parametric curves, as being easy to adjust and control [2].

Circular cylindrical pressure or vacuum vessels can be closed with help of end by different types of closures. Most pressure and vacuum vessels are closed by convex torispherical, ellipsoidal, or hemispherical heads. Geometry for torispherical head is depending by two different meridional curvatures. Hence, such domes are called two-arc vessel heads. On the other hand, ellipsoidal and hemispherical domes are called one-arc heads.
Shape of middle surface as well as thickness of the closure can different form prescribed in the standards. Smoothness in axially symmetric shell under uniform internal or external pressure can be considered as structured in the membrane stress. Junction between cylinder shell and head is discontinuity of meridional curvature. In additional for example, for a torispherical head, there is an internal discontinuity of curvatures between spherical and knuckle parts. These discontinuities of curvatures and possible discontinuity of the wall-thickness disturb the membrane stress state in pressure vessels and influence the strength of such type of structures. Vessel heads should be treated in a strength analysis as axially symmetrical shells loaded by an internal pressure or external pressure.

Elimination of a bending state in shells, we have to work towards optimization. It results in uniform stress distribution across the wall thickness. Therefore, our aimed at defining such profiles curvature of meridians curve for which the bending state in the entire structure is eliminated or it is minimized. Here we are considering two types of optimization problems: single arc meridional closures and two arc meridional closures [3]The shape of end dome of a cylindrical vessel significantly aspects the pattern of stress arising along its meridian, since the stress depends on the meridian curvature, its radius of curvature should be continuous. Curvature of commonly used torispherical sudden variation in the contact point of the knuckle and crown parts of the meridian, and in ellipsoidal heads undergoes sudden variation in the contact point of the head and cylindrical parts of the meridian. This is due to the fact that the meridional curvature radius of the cylindrical part is equal to infinity, while the further course of the meridian belonging to the head is of a finite radius. In order to avoid such a situation, the head profile should begin from the infinite radius too. [4]

II. PIONEERS OF SHAPE OPTIMIZATION AND STRESS REDUCTION

Optimization problems of such shells in the membrane stress state were discussed by Biezeno (1922) [6] ,shape of a dome with a constant thickness by Struble (1956) [7], Krużelecki (1979)[8], Torispherical pressure vessel closures were optimized by Middleton (1979) [9], Batchelor and Taylor (1995) and Błachut (1992), Discontinuity of curvatures at the junction between a cylindrical shell and a dome, similarly to internal discontinuities between different shaped parts of a dome head cause some stress concentration in the vicinity of points of discontinuities. Such problems were investigated by Magnucki et al. (2002) [12], Yushan and Wang (1996) [13] and Yushan et al. (1996) [13]. Magnucki and Lewi´nski (2000) [14] analyzed the stress state in a non-standard torispherical head composed of polynomial and circular parts.

The optimal shapes, which reduce bending effects under the assumption of constant wall thickness, were sought by Lewi´nski and Magnucki (2010) [15]. Carbonari et al. (2011) [17] discuss shape optimization of axisymmetric pressure vessels (Compressed Natural Gas tanks) considering an integrated approach in which the entire pressure vessel model is used in conjunction with a multi-objective function that aims to minimize the equivalent Huber-MisesHencky stress from nozzle to head.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

III. DISCONTINUITY EFFECT IN MERIDIONAL CURVATURE IN TORISPHERICAL SHAPE

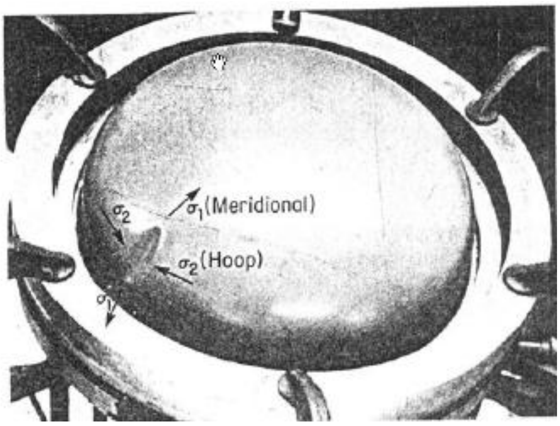

Fig:1 Defect in Head meridional [21]

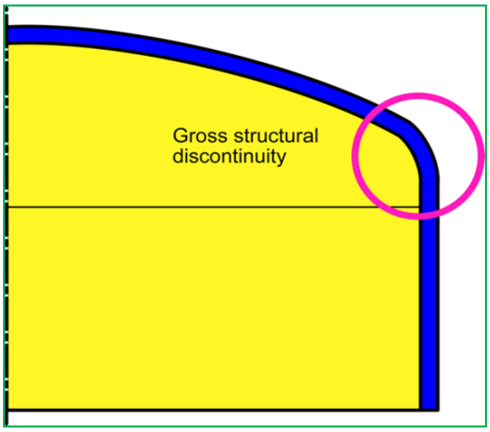

Fig:2 Gross Structural Discontinuity

IV. MATHEMATICAL FORMULATION FOR DISCONTINUITY & OPTIMIZATION

Elimination or minimize bending state in shell and dome, with respect to mode of loading and support constitute a small step toward of optimization of problem. It results in uniform stress distribution across wall thickness. Therefore, this paper is aimed to develop such a dome profile of meridians for which the stress are reducing between knuckle and crown joint, and develop smooth meridian of entire dome structure. [15].

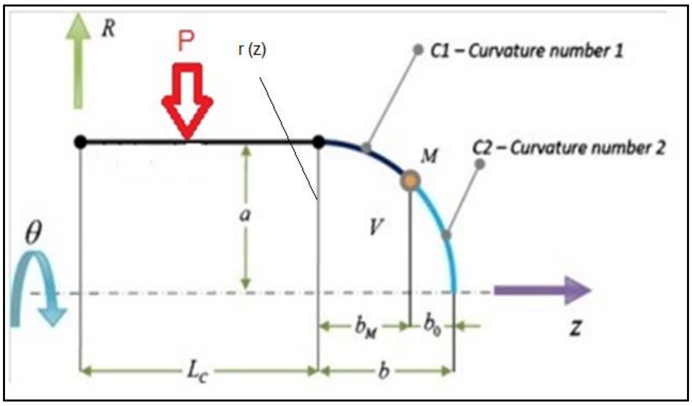

Fig. 2 Vessel loaded by an External Pressure (P), [17]

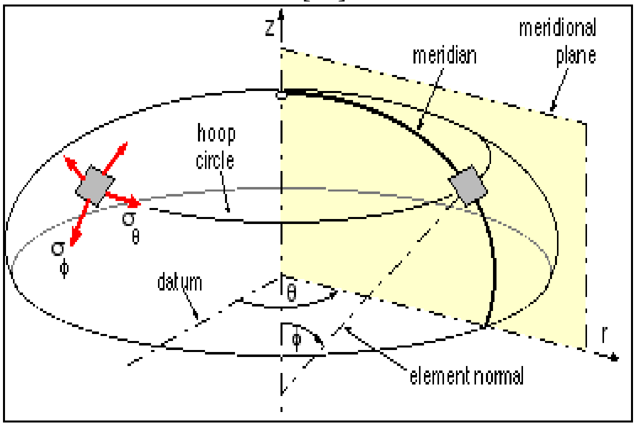

Fig.3 3D-view to visualize different parameter of dome, [18]

A. Objective function:

Nature of Design Variable:

$$X = \begin{Bmatrix} x1 \\ x2 \end{Bmatrix} = \begin{Bmatrix} bM = b \\ bo \end{Bmatrix} \qquad (1)$$

Function:

$$f(X) = \{x1\} = \{bM = b\} = \text{Maximization} \qquad (2)$$

$$f(X) = \{x2\} = \{bo\} = \text{Minimizes} \qquad (3)$$

There are some dimensionless quantities:

$\xi = z/bM$, $\beta = bm / a$, $\xi_0 = z_0 / bm$, $\beta 0 = b_0 / a$,

Where in fig 1, bM denote a part of depth (unknown) of a vessel dome and it is define by C1 curvature (for torispherical C1 = Knuckle and for $\xi = 1$ the curve C1 is ended), bo is a part of remaining depth of dome which is C2 curvature (for torispherical C2 = crown). Our objective is that function only (C1) bo = 0, which leads to $\beta 0 = 0$, and β represent the whole depth of a closure.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

B. Proposed Mathematical Model

There is a discontinuity at junction between cylindrical shell and vessel head, namely a radius of meridian Rϕ for cylinder part it is tend to infinite (Rϕ = ∞), and radius of meridian for dome is finite one, because of this difference there are some disturbances of the membrane state in a structure. To avoid this effect, we assume that meridian radius of dome ay its end point is infinite (Junction point at cylinder shell and dome (Z = 0)), the condition of equality of the meridional radii at the junction lead to the following of optimal vessel closure

$$R_\varphi^{cyl} = R_\varphi(z=0) \qquad (3)$$

$$R = a \quad for\, z = 0 \qquad (4)$$

$$\frac{dR}{dz} = 0 \quad for\, z = 0 \qquad (4)$$

$$\frac{d^2R}{dz^2} = 0\ for\, z = 0 \qquad (5)$$

Note: C^0= *Position (Joining) Continuity*, C^1 = *Slope (tangent) Continuity*, C^2 = *Curvature Continuity*

Above equitation ensure the smoothness of the meridional and circumferential curvature for z = 0

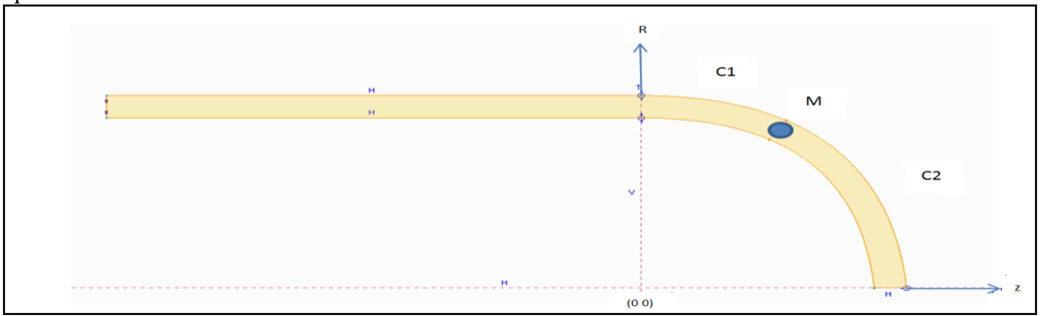

Fig 4: Junction Point (M), between crown (C2) and knuckle curve (C1)

In the case of two-arc dome (torispherical), at junction (Point M) of crown and knuckle (C1 = knuckle curvature, C2 = crown curvature), continuity of the meridional curvature are demand. It is lead to following continuity condition.

$$Rc1 = Rc2 \quad for\, z = z_M \qquad (6)$$

$$\frac{dR_{C1}}{dz} = \frac{dR_{c2}}{dz} \quad for\, z = z_M \qquad (7)$$

$$\frac{d^2R_{C1}}{dz^2} = \frac{d^2R_{c2}}{dz^2} for\, z = z_M \qquad (8)$$

C. Approximation of meridian with help of Bezier curve of 5th degree of polynomial

- Mathematically a parametric Bèzier curve is defined by [20]

$$P\,(t) = \sum_{i=0}^{n} B_i J_{n,\,i^{(t)}} \qquad (9)$$

Where, 0 ≤ t ≤ 1,

– Blending Function :

$$J_{n,i^{(t)}} = \binom{n}{i} t^i (1-t)^{(n-i)} \qquad (10)$$

$$\binom{n}{i} = \frac{n\,!}{i\,!\,(n-i)!} \qquad (11)$$

5^{th} degree of polynomial:

- Here, n = 5, since there are six vertices,

$$J_{n,i^{(t)}} = \binom{n}{i} t^i (1-t)^{(n-i)}$$

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

$$\binom{n}{i} = \frac{n\,!}{i\,!\,(n-i)!} = \binom{5}{i} = \frac{120}{i!\,(5-i)!}$$

$$J_{5,0}(t) = (1)t^0(1-t)^5 = (1-t)^5$$

$$J_{5,1}(t) = 5t(1-t)^4$$

$$J_{5,2}(t) = 10t^2(1-t)^3$$

$$J_{5,3}(t) = 5t^3(1-t)^2$$

$$J_{5,4}(t) = 5t^4(1-t)$$

$$J_{5,5} = t^5$$

D. **Governing Equation:**

$$P\ (t) = B_0\ (1-t)^5 + B_1\ 5t(1-t)^4 + B_2\ 10t^2(1-t)^3 + B_3\ 5t^3(1-t)^2 + B_4 5t^4(1-t) + B_5 t^5 \qquad (12)$$

$$rc = [t^5\ \ t^4 t^3 t^2\ \ t\ \ 1] \times \begin{bmatrix} -1 & 5 & -10 & 10 & -5 & 1 \\ 5 & -20 & 30 & -20 & 5 & 0 \\ -10 & 30 & -30 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} B0 \\ B1 \\ B2 \\ B3 \\ B4 \\ B5 \end{bmatrix} \qquad (13)$$

Where,
B0, B1. B5 are co-ordinates of the control point ($B_i \leq 1$),
t = weighting value,
$0 \leq t \leq 1$ and t = 1 denotes the end of the curvature. [17]

Dimension Dome:

Height of Dome: 493 mm	Diameter of Dome: 2680 mm	Thickness of Dome: 9 mm
Straightness of dome: 30 mm	Crown radius: 2677.16 mm	Knuckle radius: 165.5 mm

Material Use:
ASTM A-240 SS 304L

E. **This optimization co-ordinate and weighting value are following from Monte-Carlo method in following stage**

Table 1: Co-ordinates of control point and weights of the optimal shape of head meridian [19]

Point No	Coordinates		Weight Value (t)
	X (B)	Y (B)	
0	0	1	1
1	0.05718	1	0.7337
2	0.36234	1	0.9156
3	0.42733	0.67043	0.8447
4	0.56	0.39125	0.9651
5	0.56	0	0.9395

Table 2: Co-ordinates of points, from which Bezier curve getting passed

Point No	Coordinate		Weighting value
	X	Y	
0	493	0	1
1	433.75	611.61	0.7337
2	485.60	209.66	0.9156
3	470.19	373.41	0.8447

4	491.67	88.90	0.9651
5	489.12	152.07	0.9395

V. ANALYTICAL RESULTS

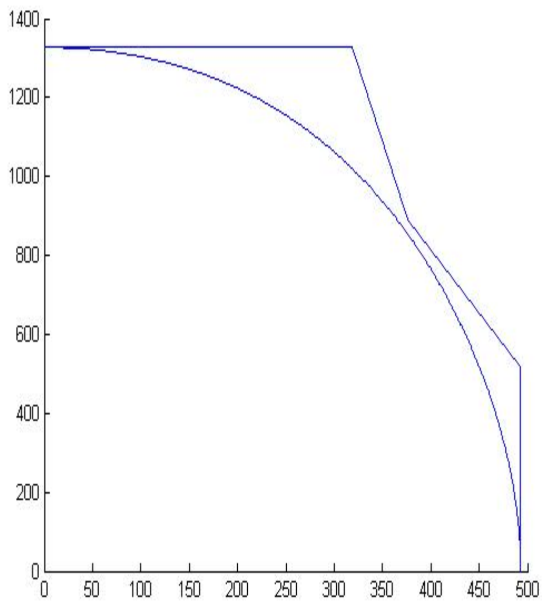

Fig:5 Validation of Result with Matlab

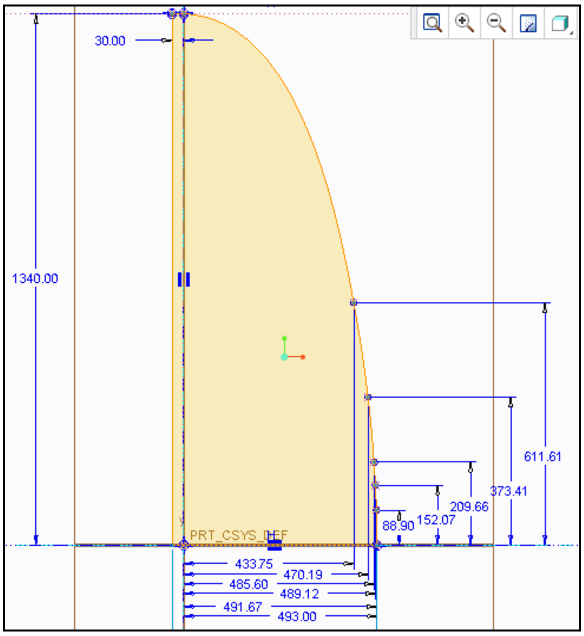

Fig:6 Shape of Bezier curve for dome meridian

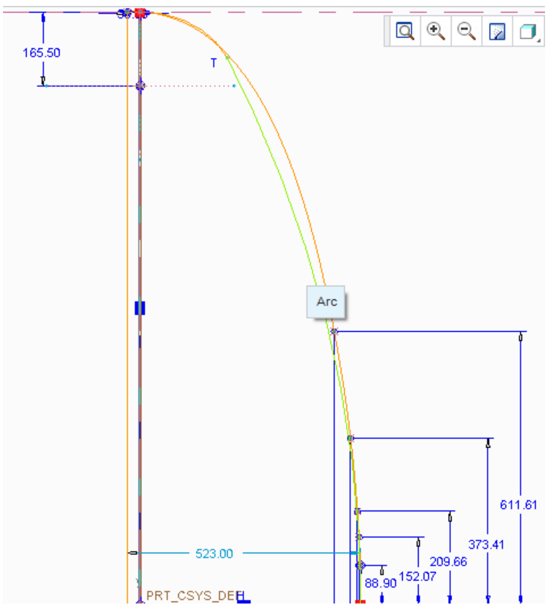

Fig 7: Arc by Torispherical dome for crown of dome

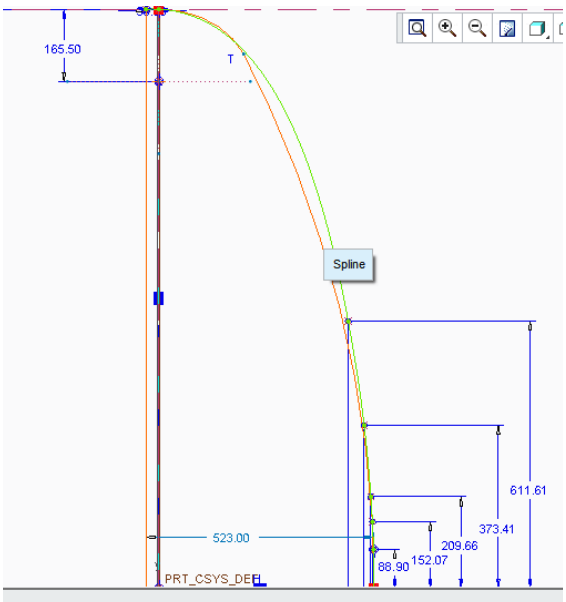

Fig 8: Spline by Bèzier curve to increase smoothness for crown of dome

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

Fig 9: Arc by original dimension for knuckle of dome

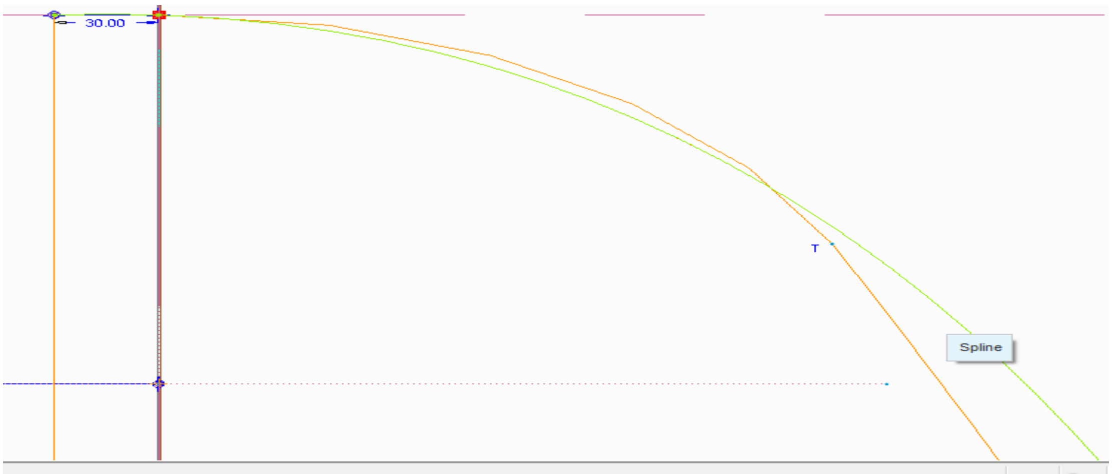

Fig 10: Spline by Bèzier curve to increase smoothness for knuckle of dome

VI. FEA RESULTS

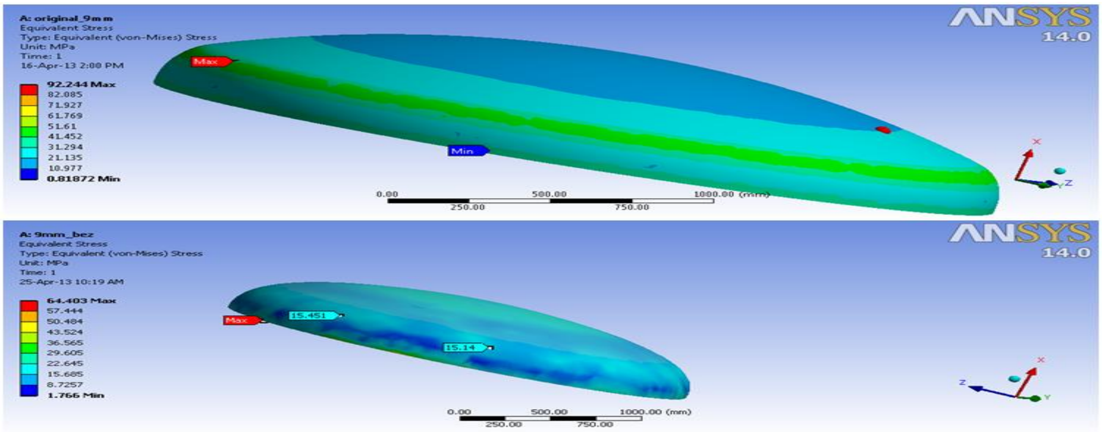

Fig 11: Stress Analysis for 9mm thickness of dome for torispherical and Bèzier dome

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

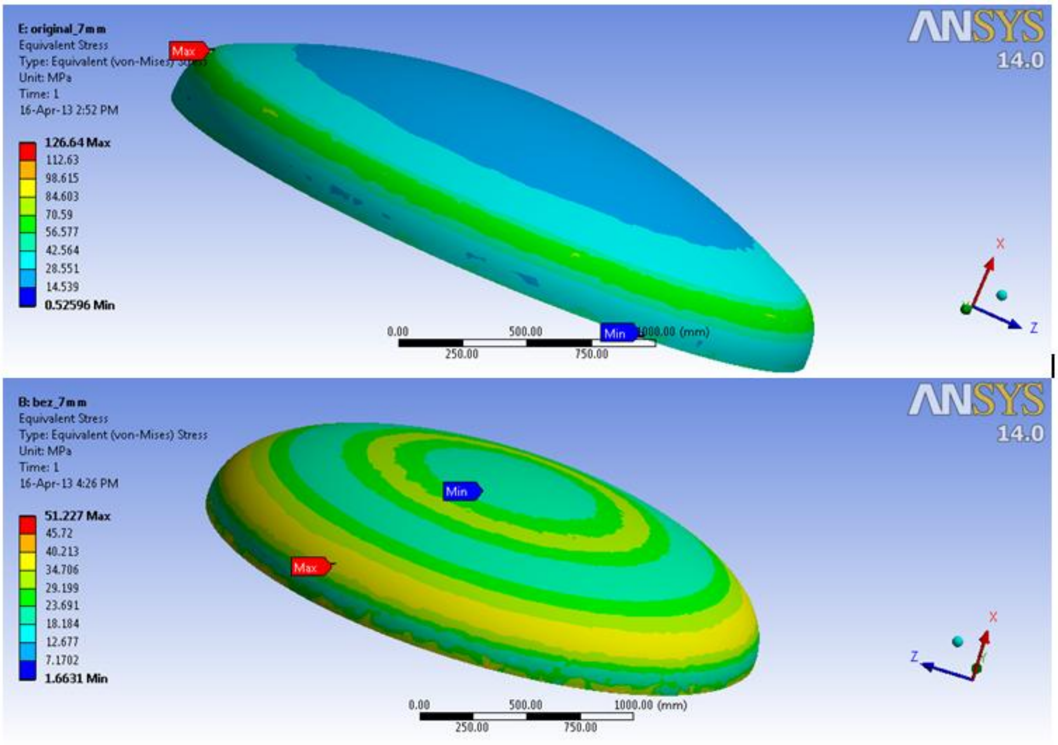

Fig 12: Stress Analysis for 7mm thickness of dome for torispherical and Bezier dome

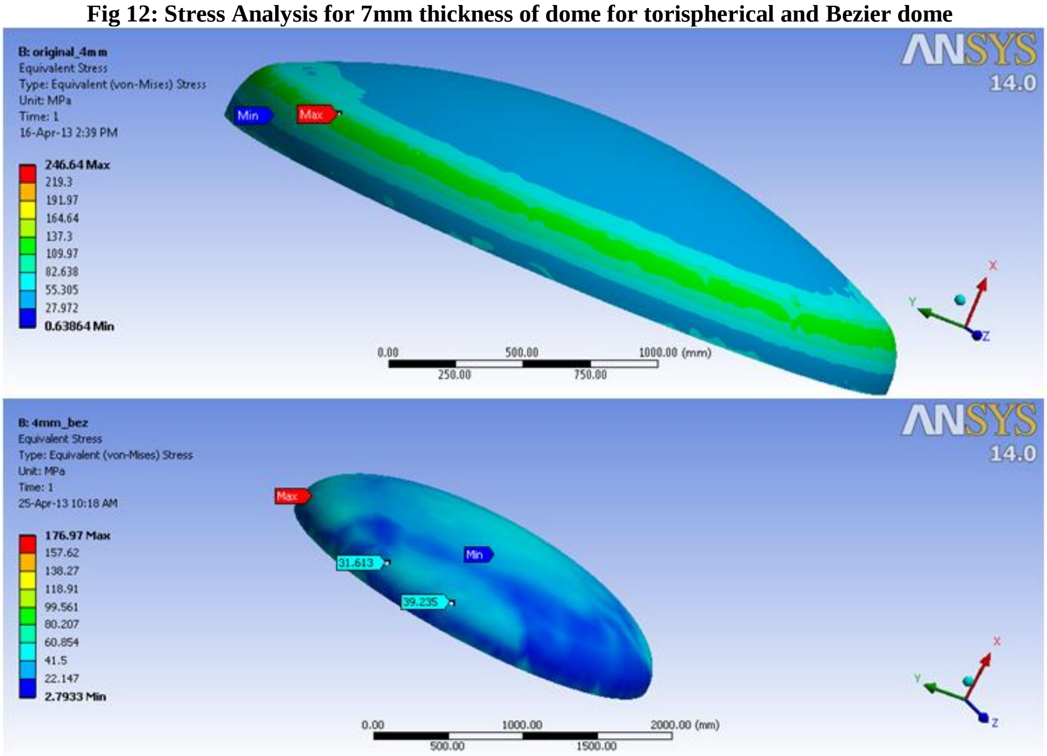

Fig 13: Stress Analysis for 4mm thickness of dome for torispherical and Bezier dome

Table 3: Result table for Torispherical dome

Sr No	Thickness (mm)	Max. stress (Mpa)	Weight (kg)
1	4	246.64	234
2	5	201.99	295

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

3	6	158.49	348
4	7	128.64	408
5	9	92.24	554

Table 4: As per Bezier dimension

Sr No	Thickness (mm)	Max. stress (Mpa)	Weight (kg)
1	3	168.74	174
2	3.5	152.73	203
3	4	105.26	231
4	5	69.82	290
5	6	59.093	345
6	7	51.22	406
7	9	37.24	551

A. Comparison Table

Allowable stress for SS 304L (Approx.) = 100 Mpa

Parameter for Comparison	Torispherical Design	Bezier Curve Design	Difference (%)
Wall Thickness (mm)	9	4	42 (Reduction)
Weight (kg)	554	231	42.46 (Reduction)

VII. CONCLUSION AND DISCUSSION

The present numerical and FEA analysis stress state of end dome of vessel enable following Conclusion:

- In torispherical design of dome, we got 92 Mpa Maximum Stress at 9 mm wall thickness of dome and also getting 554 kg of weight.
- If we reduce thickness up to 4mm wall thickness of dome without help of modern CAD tool like Bezier curve for dome meridian profile, we getting 246.64 Mpa stress and weight reduction up to 234 kg, that's mean we increase stress 37.30 %, and we reduce weight up to 42.46 %.
- If we apply Bezier curve of 5^{th} degree of polynomial with C^2 level of continuity to design of dome meridian, we getting some surprising result, at wall thickness 9mm, we getting maximum 37.24 Mpa Stress, and wall thickness reduce up to 4 mm, we increase stress 105.26 Mpa it means 35.37 %, and weight reduction up to 41.69 %.

REFERENCES

[1] BehzadAbdi, a, Hamid Mozafari,b, AmranAyob "Find the Optimum Shape Design of Externally Pressurized Torispherical Dome Ends Based on Buckling Pressure by Using Imperialist Competitive Algorithm and Genetic Algorithm Applied Mechanics and Materials Vols. 110-116.

[2] J. Lewinski, K. Magnucki,"Optimal shaping of middle surface of a dished head of circular cylindrical pressure vessel with the help of Bezier curve, EngOpt 2012 3rd International Conference on Engineering Optimization.

[3] JacekKruelecki and RafaProszowski."Shape and thickness optimization of thin-walled pressure vessel end closures, EngOpt 2012 3rd International Conference on Engineering Optimization.

[4] JacekKruzelecki, RafaProszowski,"Shape optimization of thin-walled pressure vessel end closures, Struct Multidisc Optim (2012) 46:739754.

[5] Kanagasabapathy, H.; Chockalingam, K.S.K. "Estimation of stresses at knuckle of variable thickness in torispherical heads." International Journal of Applied Engineering Research, May 1, 2009.

[6] Biezeno CB (1922) Bijdrage tot de berekening van katelfronten. De Ingenieur 37.

[7] Struble RA (1956) Biezeno pressure vessel heads. J ApplMech 23:642–645.

[8] Krużelecki J (1979) some problems of optima design of axially symmetrical shells in membrane state. MechTeorStosow 17(1):75–92 (in Polish).

[9] Middleton J (1979) optimal design of torispherical pressure vessel end closures. EngOptim 4:129–138.

[10] Batchelor MJ, Taylor TE (1995) Equivalent torispherical pressure vessel heads. Int J Press Vessel Pip 7(3):229–244.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 3, May 2013

[11] Błachut J (1992) Influence of meridional shaping on the collapse strength of FRP domes. EngOptim 19:65–80.

[12] Magnucki K, Szyc W, Lewi´nski J (2002) Minimization of stress concentration factor in cylindrical pressure vessels with ellipsoidal heads. Int J Press Vessel Pip 79:841–846.

[13] Yushan Z, Wang ZR (1996) Stress analysis of two-arc approximate ellipsoidal pressure vessel heads and parameter optimization. Int J Press Vessel Pip 67:199–202.

[14] Magnucki K, Lewi´nski J (2000) fully stressed head of a pressure vessel. Thin-Walled Struct 38:167–178.

[15] Lewi´nski J, Magnucki K (2010) Shaping of a middle surface of a dished head of a circular cylindrical pressure vessels. J TheorApplMech 48(2):297–307.

[16] Carbonari RC, Muñoz-Rojas PA, Andrade EQ, Paulino GH, Nishimoto K, Silva ECN (2011) Design of pressure vessels using shape optimization: an integrated approach. Int J Press Vessels Pip 88:198–212.

[17] JacekKrużelecki, RafalProszowski , "Shape optimization of thin-walled pressure vessel end closures", Struct Multidisc Optim (2012) 46:739-754.

[18] http://wwwmdp.eng.cam.ac.uk/web/library/enginfo/textbooks_dvd_only/DAN/pressVessels/shells/shells.html.

[19] J. Lewinski, K.Manguncki, " Optimal Shaping of middle surface of a dished head of circular cylinder pressure vessel with help of Bezier curve", EngOpt 2012- 3rd International Conference on Engineering Optimization.

[20] David F. Rogers, J. Alan Adams, "Mathematical Elements for Computer Graphics" Second Edition, p-291-292.

[21] http://www.pveng.com/ASME/ASMEComment/FandDHeadStress/FandDHeadStress.php

AUTHOR BIOGRAPHY

Jignesh Dipakkumar Jani.[1]
B.E. Mechanical, M.Tech – Mechanical (CAD/CAM)
Mechanical Engineering Department,
Nirma University, Ahmedabad, 382481

Manoj Kumar Gupta[2]
B.Sc– Engineering, M.E. – Mechanical (Cryogenics)
Engineer-SE
Cryogenics Division,
Institute for Plasma Research, Bhat-382428, Gandhinagar, India.

Reena R. Trivedi[3]
B.E. Mechanical, M.Tech- Mechanical (CAD/CAM)
Sr. Associate Professor,
Mechanical Engineering Department, Nirma University, Ahmedabad, 382481