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A Review: Curvature Approximation on Triangular Meshes

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Abstract*-This paper reviews the previously available methods to estimate the curvatures, both discrete and continuous for triangulated surface. Many applications such as surface segmentation, surface smoothing, surface reconstruction, and registration require the accurate estimation of curvature at the vertices of the triangulated surface. For the estimation of the curvature, several different schemes have been proposed which are discussed in the literature. We examine implementation procedures for each technique and discuss its advantages and disadvantages in terms of accuracy, cost and time. Some recent research results are also included and discussed in this paper.

***Keywords* - Curvature Estimation, Gaussian Curvature, Mesh, Triangulated Surface.**

I. INTRODUCTION

In computer graphics, curvature is a fundamental property as it provides information on local shape of a surface. Curvature as a first step, facilitates many surface processing tasks, such as surface segmentation[1]-[2], surface smoothing[4]-[46], surface reconstruction[6] and registration[15]-[35]. Since a triangle mesh is a piecewise linear surface, curvature has to be estimated. In recent studies, curvature as a fundamental descriptor for shape analysis was an important parameter for solving a variety of basic tasks in computer graphics, including surface classification[3], matching[5] , re-meshing[7]-[8] , symmetry detection[9] , non-photorealistic rendering[10]-[11], and feature line extraction[12]-[13]. Some elementary differential geometry properties such as normal and curvatures are as important as surface positions for the perception and understanding of shapes.

The curvature of a curve is the measure of its deviation from a straight line in a neighborhood of a given point, and the curvature becomes greater as this deviation becomes greater [34]-[55](Fig.1). First, we review the basic mathematics behind curvature calculation—

Let $r(u,v)$ be a regular surface S. The point p lies on the surface. The unit normal vector field of $r(u,v)$ is defined by [29]-[34]

$$N(u,v) = \frac{r_u(u,v) \times r_v(u,v)}{\|r_u(u,v) \times r_v(u,v)\|}$$

where $(u, v) = \frac{\partial r(u,v)}{\partial u}$, $r_v(u,v) = \frac{\partial r(u,v)}{\partial v}$

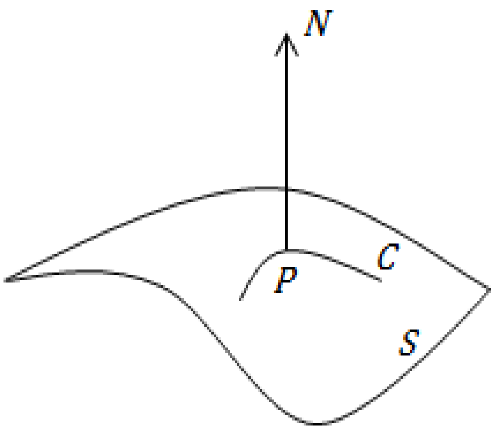

Fig.1: A curve C on the surface S having a point P with unit normal N

Meusnier's theorem [29]-[34]:

Normal curvature of curve C, at P is $k_t = k.\cos\theta$ for curvature of C, at S and θ is the angle between normal vector of surface curvature n and surface normal N of S. The principal curvatures k_1 and k_2 of S are defined as the maximum

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and minimum normal curvatures, respectively. The directions corresponding to these normal curvatures are called principal directions.

Euler's theorem [29]-[34]:
The Euler theorem gives the relationship between the normal curvature k_T of an arbitrary normal section T and principal curvatures k_1 and k_2

$$k_t = k_1 cos^2\emptyset + k_2 sin^2\emptyset$$

where, ϕ is the angle between T and T_1.
The total curvature and mean curvature are uniquely defined by the principal curvatures of the surface

$$K = k_1 . k_2 \text{and} \qquad H = \frac{k_1 + k_2}{2}$$

The techniques for curvature estimation on triangle meshes can be broadly classified into two categories -

1. Continuous method
2. Discrete method

In the first method, a parametric surface patch is fitted to the neighborhood of each set of data points using surface fitting. Then the curvatures are computed by interrogating the fitted surface. This method of approximating curvature is very popular. The discrete method is used for direct approximation of the curvature or curvature directly on the discrete representation of the underlying surface that generated the triangle mesh from which Gaussian curvature, normal curvature and other differential properties can be estimated.

II. SURFACE PATCH FITTING METHODS

A. Quadric Fitting Method

Hamann (1993) [14] proposed a technique which fits a quadric approximated locally to a neighborhood of vertices and then curvatures are computed by interrogating the quadric. The approximated surface is computed using least square method which can locally be represented as a graph of a bivariate function. It is independent of the two unit vectors determining a local orthonormal coordinate system. The principal curvatures at a point on the graph of such a quadratic polynomial are used as the principal curvatures at an original surface point. In this paper, the test example chosen are all graphs of bivariate functions of leading to an obvious error measure.
Krsek (1997)*et al.* [31] describes a method for estimation of principal curvatures and principal directions from 3D scanned data which acts as a base for the extracting intrinsic curves on the surface. There are two steps for estimation of differential parameters which are intended for matching partially overlapping range images. First step approximates a small neighborhood of a vertex by an osculating paraboloid. The differential parameters of the paraboloid give an initial estimate of the parameters of the original surface. Then using estimated parameters, these initial estimate is further refined from the neighborhood of each point.
Stokely (1992)*et al.* [33] presented and characterized five practical solution for the difficulty of curvature sampling of arbitrary fully described 3D objects. One of these methods is always based on a proper surface parameterization, called cross patch method. It is shown to be very fast and robust in presence of noise. In this method he considered two orthogonal traces u and v on the underlying surface intersecting at the inspection point. Since all the required partial derivatives after the fitting traces can be calculated. Gaussian and mean curvatures can be determined since this method requires fewer sample points, it reduces the computational load.
Goldfeather and Interrante (2004) [17] developed a cubic-order approximation method which shows variation of the quadratic fitting method for the approximation of curvature. In this method, neighboring points and corresponding normal vectors at adjacent vertices are used to create third degree terms in the least-squares solution for surface fitting. They described and then applied three different methods which approximate the Weingarten curvature matrix at a vertex of the mesh of test surface and examined the directional errors.One of three methods is the Adjacent-Normal Cubic Approximation Method in which surface is fit to the adjacent vertex data.

$$f(x,y) = \frac{A}{2}x^2 + Bxy + \frac{c}{2}y^2 + Dx^3 + Ex^2y + Fxy^2 + Gy^3$$

Values of (A, B, C) can be obtained by using third degree term in the least-squares fit for different form.

B. Circular fitting method

Chen and Schmitt (1992)[30] described an algorithm to estimate the principal curvatures at the vertices of triangulated surface by using Meusnier's theorem and Euler's theorem.
Normal curvature of curve C, at P by Meusnier theorem is

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$$k_t = k.\cos\theta$$

A relationship between the normal curvature k_T of an arbitrary normal section T and principal curvatures k_1 and k_2 is drawn by Euler's formula

$$k_t = k_1 \cos^2\emptyset + k_2 \sin^2\emptyset$$

Where $\emptyset$ is the angel between T and T_1 (Fig.2).

They define the no of surfaces passes through P by forming n vertex triples (P, P_i, P_j) where P_i , P_j are two geometrically opposite vertices with respect to P in its neighborhood. In this method, a circum-circle through these vertices is drawn. A circle passing through each triple of vertices as an approximation of the surface curves.

C. Bezier patch method

Razdan *et al*.[16] used a method with biquadratic Bezier patches as a local surface fitting technique for the approximation of curvature. In this paper, biquadratic Bezier surface patch is used for approximation of the neighborhood of mesh vertex for computation of curvature instead of taking the quadric analytical function approach. A biquadratic Bezier surface [47] is written as

$$x(u,v) = \sum_{i=0}^{2}\sum_{j=0}^{2} b_{i,j} B_i^2(u) B_j^2(v)\,; \qquad u,v \in [0,1]$$

Where $b_{i,j}$ are called Bezier *control points* which can be computed by a standard least squares fit to a set of vertices on the surface [47]-[37].

Fig. 3, shows a biquadratic Bezier patch fitted to a set of points (P_i).

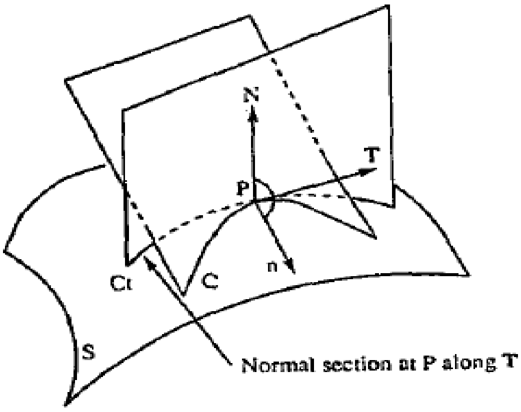

Fig.2: Local surface geometry around a point P[30]

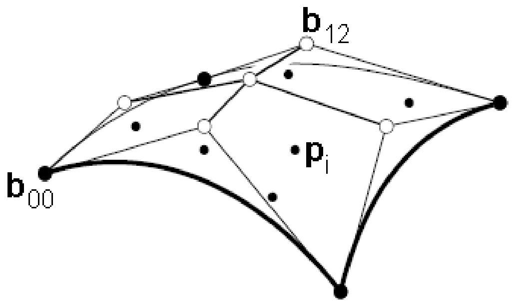

Fig.3: a biquadratic Bezier patch fitted to data points P_i [16]

After fitting the vertices on surface, a linear equation system $Ax = B$ is built where A is computed from the Bernstein basis function contribution of each point, x is single column matrix of unknown control points $b_{i,j}$ and B is the single column matrix of n points to be fitted.

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$$\begin{bmatrix} B_0^2(u_0)B_0^2(v_0) & B_0^2(u_0)B_1^2(v_0) & \cdots & B_2^2(u_0)B_2^2(v_0) \\ B_0^2(u_1)B_0^2(v_1) & B_0^2(u_1)B_1^2(v_1) & \cdots & B_2^2(u_1)B_2^2(v_1) \\ B_0^2(u_2)B_0^2(v_2) & B_0^2(u_2)B_1^2(v_2) & \cdots & B_2^2(u_2)B_2^2(v_2) \\ & & \vdots & \\ B_0^2(u_n)B_0^2(v_n) & B_0^2(u_n)B_1^2(v_n) & \cdots & B_2^2(u_n)B_2^2(v_n) \end{bmatrix} \begin{bmatrix} b_{0,0} \\ b_{0,1} \\ \vdots \\ b_{2,2} \end{bmatrix} = \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$$

where $x = [b_{0,0} \quad b_{0,1} \cdots \quad b_{2,2}]^T$ and $B = [p_0 \quad p_1 \cdots \quad p_n]^T$

From this system of equations, x is computedto determine the control points b_{ij}, which construct the biquadratic Bezier surface. An adjusting matrix and smoothing constraint are employed to modify the system as follows

$$\begin{bmatrix} \alpha A \\ (1-\alpha)S \end{bmatrix}[x] = \begin{bmatrix} \alpha B \\ 0 \end{bmatrix}$$

This method has more degrees of freedom and therefore the surface fit is more flexible to other.
An extended Bezier biquadratic patch to third order fitting patch for surface fitting to estimate the curvature, is used by **Li Z(2009)*et al.*[48]**. A third order fitting patch is applied for mesh smoothing. A bi-cubic Bezier surface is written as

$$B(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} B_{i,3}(u)B_{j,3}(v)b_{ij}\ ,\ u,v \in [0,1]$$

Fig.4,shows weighted bi-cubic Bezier surface to fit the vertex *pi* and its 2-ring neighborhood.

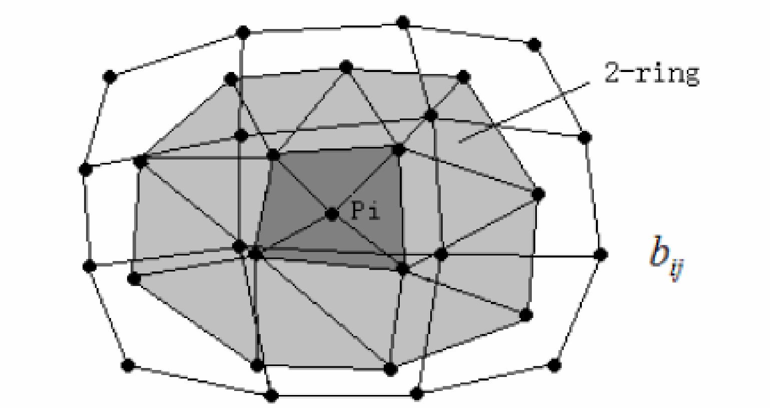

Fig.4: A vertex and its two ring neighborhood is fitted by bi-cubic Bezier surface [48]

The parameters of each vertex are denoted by (u_i, v_i)which have to be computed first. After getting the corresponding parameters of vertex p_i and other fitting vertices, the author considered linear equation system Ax=B, where

$$A = \begin{bmatrix} B_0^3(u_0)B_0^3(v_0) & B_0^3(u_0)B_1^3(v_0) & \cdots & B_3^3(u_0)B_3^3(v_0) \\ B_0^3(u_1)B_0^3(v_1) & B_0^3(u_1)B_1^3(v_1) & \cdots & B_3^3(u_1)B_3^3(v_1) \\ & & \vdots & \\ B_0^3(u_n)B_0^3(v_n) & B_0^3(u_n)B_1^3(v_n) & \cdots & B_3^3(u_n)B_3^3(v_n) \end{bmatrix}$$

From above system, Bezier control points b_{ij} can be calculated which construct the bi-cubic Bezier surface patch. To construct the patch more effective, an adjusting matrix and a factor as used in [16] is added to modify the system. This weighted bi-cubic Bezier patch more accurately obtains the normal vector and curvature estimation of the mesh model compared to other algorithms. The experiments show that it obtains satisfactory results.

D. Normal curvature based method

Taubin (1995) [21], estimated the curvature tensor of surface at the vertices in a polyhedral approximation. Taubin proposed an algorithm to compute principal curvatures and principal directions at a vertex of a triangle

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mesh. The principal curvature directions as two eigen vectors of the symmetric matrix and principal curvatures as linear combinations of the two eigenvalues of the same matrix are obtained. This method is advantageous because of its simplicity. The directional curvature function $\kappa_p(T)$is a quadratic form [29]-[42], i.e. it satisfies the identity

$$\kappa_p(T) = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}^t \begin{pmatrix} \kappa_p^{11} & \kappa_p^{12} \\ \kappa_p^{21} & \kappa_p^{22} \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$$

where $T = t_1T_1 + t_2T_2$ is a tangent vector to Sat p,{ $T_1; T_2$} is an orthonormal basis of the tangent space to Sat p and $\kappa_p^{11} = \kappa_p(T_1)$, $\kappa_p^{22} = \kappa_p(T_2)$, $\kappa_p^{12} = \kappa_p^{21}$. The vectors $\{T1;\ T2\}$ are called principal directions of S at p when $\kappa_p^{12} = \kappa_p^{21} = 0$.

A matrix Mp is defined by an integral formula which has the same eigenvectors as Kp, and their eigenvalues are related by a fixed homogeneous linear transformation. Let T_θ is the unit length tangent vector for $T_\theta = \cos(\theta)\, T_1 + \sin(\theta) T_2$, where $-\pi \le \theta \le \pi$ and $\{T_1, T_2\}$ are the orthonormal principal direction. Now a symmetric matrix is defined

$$M_p = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \kappa_p(T_\theta) T_\theta T_\theta{}^t d\theta$$

where$\kappa_p(T_\theta) = \kappa_p{}^1 cos^2(\theta) + \kappa_p{}^2 sin^2(\theta)$

Taubin proved that above symmetric matrix can be factorized as

$$M_p = T_{12}^t \begin{pmatrix} m_p^{11} & m_p^{12} \\ m_p^{21} & m_p^{22} \end{pmatrix}$$

The complexity of this algorithm is linear, both in time and in space. Experiment shows the better results than other methods.

Surazhsky (2003)*et al.* [41] suggests two modifications to Taubin's method. One is to replace weighted areas of the incident triangles by weighted incident angles for computing the normal at a vertex and the second is to replace the directional curvature by average of curvatures at the two vertices of an edge.

Theisel (2004)*et al.* [24]presented a method called the *normal based estimation of the curvature tensor for Triangular meshes*. Instead of each vertex, curvature is approximated for a single triangle in this method. The curvature tensor of a smooth surface can be defined as first order partials of points on surface and their normal vectors. In this method, two linear interpolations of vertices and their normal vectors of a triangle are performed to calculate the curvature tensor on a single triangle. This method gives satisfactory results for meshes with regular triangulation. This method apply finite differences to first estimate normal curvatures along edges then the curvature tensor is calculated either directly [21] or by using least-squares fitting. Each triangle of the mesh is considered independently and calculated the curvature tensor as a smooth function on it. The basic idea for doing so is taken from the well-known concept of Phong-shading [49] in which two linear interpolations are applied on each triangle of a mesh together with its vertex normal. This linear interpolation for the vertices gives the current location of it, while the linear interpolation of the normal at vertex gives the normal for the illumination model. The normal which got from the piecewise linear surface, generally differs from the interpolated normal obtained from this method. This method produces smooth representations of mesh models. A method having finite difference approach which estimates the curvature per face on irregular triangle meshes is proposed by **Rusinkiewicz (2004)[25]**. It might be a thought of extension of method estimating per-vertex normal. The curvature at the vertex is calculated by taking a weighted average of the adjacent faces' normal vectors.Curvature per face is described by directional derivative of the surface points and their normal and differences between vertices and between their normal vectors on a face is used to calculate it.

The normal curvature k_n of a surface is the reciprocal of the radius of the circle in some direction [45] and it varies with the direction, but for a smooth surface it satisfies

$$\kappa_n = (s \quad t)\begin{pmatrix} e & f \\ f & g \end{pmatrix}\begin{pmatrix} s \\ t \end{pmatrix} = (s \quad t)\, \mathrm{II} \begin{pmatrix} s \\ t \end{pmatrix} \qquad (1)$$

The second fundamental tensor II, already seen in equation (1), is defined in terms of the directional derivatives of the surface normal

$$\mathrm{II} = (D_u n \quad D_v n) = \begin{pmatrix} \frac{\partial n}{\partial u}.u & \frac{\partial n}{\partial v}.u \\ \frac{\partial n}{\partial u}.v & \frac{\partial n}{\partial v}.v \end{pmatrix}$$

When this tensor is multiplied by any vector in the tangent plane, gives the derivative of the normal in the direction $\mathrm{II}s = D_s n$.

Zhihong2009)*et al.* [44] described a curvature estimation method for meshes in which each planer triangular facet was converted into a curved patch using the vertexpositions and the normal of three vertices of each triangle. This method calculates the per triangle curvature of the neighboring triangles of a mesh point. This method is efficient and its accuracy is comparable to others. This method specifically avoids solving a least squares problem and is able to handle arbitrary triangulations. In this method, each neighboring triangle of a mesh point is considered and it is interpolated by a triangular cubic Bezier patch. Bezier curve are defined explicitly as

$$f(t) = \sum_{i=0}^{n} b_i B_i^n(t) \text{ , where } B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

Here, the local behavior of a surface $S(u,v)$ at a single point P with a given local parameterization (u,v) is considered.

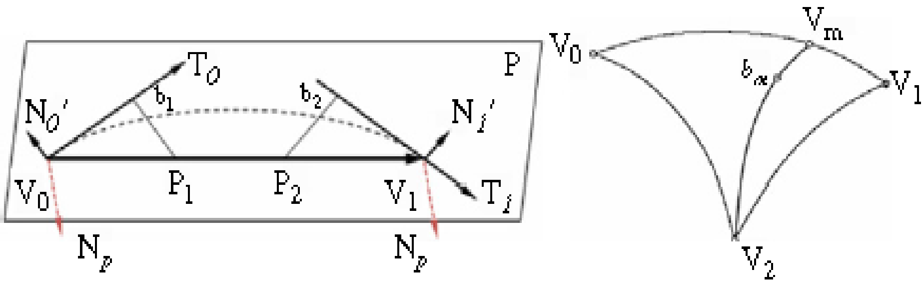

Fig.5: Construction of cubic Bezier curve and a triangular Bezier patch [44]

The principal curvature values and directions can be computed by using the Weingarten operator W, also known as the shape operator. W is the inverse of the first fundamental form multiplied by the second fundamental form

$$W = I^{-1}II = \frac{1}{EG - F^2}\begin{bmatrix} GL - FM & GM - FN \\ EM - FL & EN - FM \end{bmatrix}$$

Eigen decomposition of W (shape operator) gives the principal curvatures and value of principal curvatures is used for computing Gaussian and mean curvature. **Meek and Walton (2000) [19]** performed a technique for comparing approximations which is called asymptotic error analysis. Errors of several approximations to the surface normal and to the Gaussian curvature can be compared by using this technique. They used angle deficit method in which curvatures are estimated directly on the discrete triangle meshes on the basis of the Gauss–Bonnet theorem.

E. Gauss Bonnet scheme

Considering a vertex v and its immediate$\{v\}_{i=0}^{n-1}$. Let α_i be the angle at v between two successive edges $e_i = \overline{vv_i}$ and γ_i be the outer angle between two successive edges of neighboring vertices of v, then

$$\sum_{i=0}^{n-1} \alpha_i = \sum_{i=0}^{n-1} \gamma_i \qquad (1)$$

Then in the polygonal case, the Gauss-Bonnet theorem reduces [29]-[34] to

$$\iint_A 2\pi - \sum_{i=0}^{n-1} \gamma_i$$

By equation (1)

$$\iint_A 2\pi - \sum_{i=0}^{n-1} \alpha_i$$

Here A is the accumulated area of the triangle around v. Gaussian andmean curvature can be written as

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$$K = \frac{2\pi - \sum_{i=0}^{n-1} \alpha_i}{\frac{1}{3}A} \text{ and } \quad H = \frac{\frac{1}{4}\sum_{i=0}^{n-1} \|e_i\|.\beta_i}{\frac{1}{3}A}$$

Where β_i is determines normal deviation.
In the method [19], surface is estimated by a polyhedron with triangular faces which have vertices as points on the surface. The figure shows point $O$is surrounded by the triangular faces$P_i OP_{i+1}$. The spherical image of the polyhedron is a set of points on the unit sphere and these points on the surface can be joined by arcs to form a spherical polygon on the unit sphere.

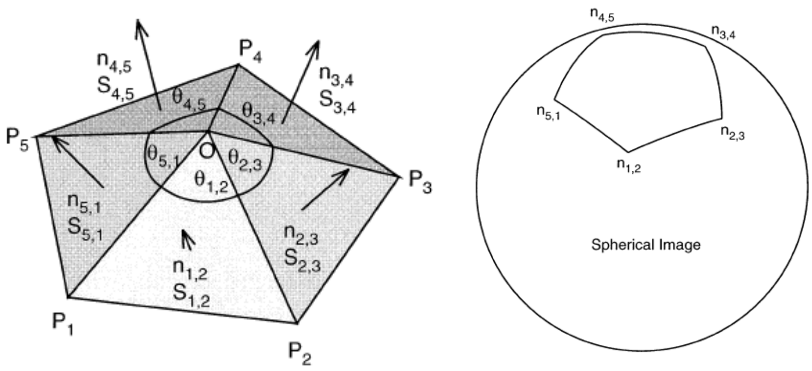

Fig.6: Angle deficit method [19]

The area of the spherical polygon is called as the angle deficit of the polyhedron which is an approximation to the curvature at O

$$k = \frac{2\pi - \sum \theta_{i,i+1}}{\frac{1}{3}\sum S_{i,i+1}}$$

Myers (2003) *et al.* [20] described discrete differential geometry operators and estimate attributes for triangle meshes. The author considered that using the 1-ring neighborhood around the vertex is sufficient for approximating curvature at the vertex on the triangle mesh.Using *averaging Voronoi cells* and the mixed Finite-Element/Finite-Volume method, Author described a consistent derivation of the first and second order differential properties for piecewise linear surfaces such as arbitrary triangle meshes. In the tangent plane, for each unit directione_θ, the normal curvature $k^N(\theta)$ is the curvature of the curve that belongs to both the surface itself and plane containing both n and e_θ . The mean curvature

$$H = \frac{1}{2\pi}\int_0^{2\pi} k^N(\theta)d\theta$$

A direct relation between surface area minimization and mean curvature flow can be described as

$$2H.n = \lim_{diam(A)\to 0} \frac{\nabla A}{A}$$

Gaussian curvature can be expressed as

$$K = \lim_{diam(A)\to 0} \frac{A^g}{A}$$

Where A^g is the spherical image (area of Gauss map image) for infinitesimal surface A.
For any triangulation, the mean curvature normal is

$$\iint_{A_M} K(x)\, dA = \nabla A_{1-ring}$$

Mesmaudi (2011)*et al.*[50] focused on concentrated curvature which further used to estimate Gaussian, mean and principal curvature. A discrete version of Gauss-Bonnet theorem is satisfied by concentrated curvature, used as an important tool for analyzing attributes of triangulated surface.

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III. DISCUSSION

We have reviewed and analyze different discrete and continuous approaches for estimation of curvature on triangulated surface. The discrete method employs a direct approximation formula for the curvature to calculate attributes of surface while continuous methods applies a surface fit locally to the vertex and some neighborhood around it. Discrete method reduces the computational cost associated with the surface fitting method because it reduces the computation time at the cost of attainable accuracy. Using parametric form of surface in place of analytical has many advantages such as flexibility of surface fit and ability to add smoothing constraints. For irregular triangulation data, Bezier method for curvature estimation always performs better than others. Normal based method works better for regularized triangulation but it consistently has large maximum error. Cubic order method proposed by Goldfeather and Interrante is more stable and performed better than normal based method. Osculating paraboloid fitting scheme always has a very good convergence which shows it is very stable. Meek and Walton proved that paraboloid fit has a quadratic error bound using asymptotic analysis even for non-uniform meshes while Gauss-Bonnet scheme has quadratic error bound for uniform mesh. Algorithm presented by Ruskinwicz is robust and free from degenerate in presence of irregular tessellation and moderate amount of noise.

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