Studies on Drag Reduction in Conical Tanks Using Polymer Additions in Gravity Driven Flow Systems

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Abstract- Drag reduction is the technique where it can help in oil and chemical industry in maximizing the flow potential of pipelines increasing operational flexibility, throughputs, capacity and hence increasing bottom line profit potentials. Present Study was carried out with a polymer additive (poly acryl amide) as a drag reducing agent to reduce the efflux time of gravity draining of a Newtonian liquid from a conical tank (where the flow is laminar) through exit pipe system (where the flow is turbulent). Equations for efflux time were also derived based on macroscopic balances, such macroscopic balances are useful for preliminary estimation of an engineering problem. About 480 experimental runs were conducted to find the efflux time for draining the conical tank of its contents (with water soluble polymer) and Polymer addition to the vessel shows significant reduction in % drag reduction. i.e., Draining of tank contents was found to be relatively faster when polymer solutions were added and also for PAM concentration of 30ppm, draining of tank solution with minimum efflux time was observed and in the presence of polymer solution, % of drag reduction was found certain by a minimum of 18% and maximum of 26 % on an average.

Key words: Drag reduction, efflux time, friction factor, polymers.

I. INTRODUCTION

In fluid flow operations, drag reduction leads to an increased efficiency of operation during its transportation. Earlier studies have identified active and passive methods for drag reduction. Drag reduction by additives was considered one among the active methods. Processing and storage vessels in the chemical and allied industries appear in a large variety of shapes. The time required to drain these vessels of their liquid content can be of crucial importance in many emergency situations or accident scenarios aside from sheer productivity considerations [1]. When vessels of such large size were to be slowly drained by gravity through an exit piping system, the flow is essentially laminar in the tank and turbulent in the pipe depending upon the viscosity of liquid and diameter of the exit pipe. Laminar flow sometimes known as streamline flow occurs when a fluid flows in parallel layers, with no disruption between the layers [2]. Drag-reducing polymers are long-chain, ultra-high molecular weight polymers. Typical molecular weights for drag-reducing polymers range from 1 to 10 million, with higher molecular weight polymers giving better drag reduction performance. With only parts-per-million levels in the pipeline fluid, drag-reducing polymers suppress the formation of turbulent bursts in the buffer region and thus suppress the formation and propagation of turbulent eddies [3]. The net result of using a drag-reducing polymer in turbulent flow is a decrease in the frictional pressure drop in the pipeline. Turbulence and the resulting frictional pressure drop were shown to be reduced by as much as 70% with drag-reducing polymers. Drag reducing polymers are used in both oil and water flow applications. Although other drag-reducing polymers are used, oil drag reducers are typically ultra-high molecular weight polymers (alpha-olefins), while water-soluble drag reducers are typically Polyacrylamide polymers. The drag reduction effect is extremely interesting from a practical point of view. Liquids are mostly transported through pipes, and drag reduction by adding a small amount of polymers can offer large economic advantages and a larger effectiveness of this transportation. In addition to drag reduction, polymer additives result in the reduction of heat losses in thermal systems which is important in maintaining low oil viscosity[4] and the reduction in heat transfer is much more pronounced[5] than reduction in friction. A similar application is the addition of polymers to oil being transported from offshore platforms to shore facilities [6]. Also, in sewage pipes and storm–water drains, polymers have been added to increase the flow rates so that the peak loads do not result in overflowing. If only relatively infrequent use of the piping system is required, this can be much cheaper than launching new pipes [7]. The addition of low concentrations of polymers might be capable of improving blood flow through stenotic
vessels without altering flow through normal vessels [8]. Polymers were also employed for reduction in friction in slurry flows [9]. Ortiz and Bessa [10] were reported that for drag reduction to take place, the flow should be either turbulent or disturbed. They also mentioned that drag-reducing polymers in disturbed flow conditions reduced the hydrodynamic resistance. Gazuko and Gorodstov [11] were reported that polymers were considered to be an effective medium for influencing the viscous sub layer and increasing its thickness. They carried out experiments in rough pipes using Polyacrylamide (PAM) & Polythene oxide (PEO) solutions since rough pipe could lead to thinning of viscous sub layer.

The present study is envisaged to obtain data on efflux time during gravity draining of a Newtonian liquid from a large conical tank (where the flow is laminar) through single-exit pipe system (When the flow is turbulent). Data were obtained both in the absence and presence of polymer additives. Polyacrylamide (PAM) was used as drag reducing agent. Polymer solutions of four different concentrations were used to study the effect of polymer concentration on draining time. PAM was used since it is water-soluble and resists shear degradation [12]. Water was chosen as the Newtonian fluid, since water was a good solvent that offered excellent resistance to shear degradation of polymer additives [13]. Experiments were performed for assessing the efflux time when water soluble polymer solutions are added to a open cylindrical tank drained by two exit pipes (The flow in each of the pipes is assumed to be turbulent). The polymer used was Polyacrylamide (PAM) and a generalized correlation for friction factor was developed. The friction factor equation reported by Bird et al [15] can be considered as generalized friction factor equation for calculating the efflux time for the polymer solutions [14].

II. DEVELOPMENT OF MATHEMATICAL EQUATION FOR EFFLUX TIME FOR A CONICAL TANK

The schematic diagram of the equipment used to develop the mathematical equation for efflux time and also to obtain the experimental data is shown in Fig.1. The Conical tank along with exit pipe is initially filled with a Newtonian liquid, i.e. water. The exit pipe is provided with a Gate valve (GV) at the bottom end and the fluid is assumed to leave the exit pipe (station-2) under the turbulent flow conditions. The height of the liquid in the tank is displayed through a level indicator (LI). It is desired to find the time required to drain the tank contents (not the pipe) mathematically.

Unsteady state mass balance across station – 1 and station – 2 can be written as

\[
\frac{d}{dt}(M_{tot}) = W_1 - W_2
\]

For the present system

\[
W_1 = 0 \text{(Since no liquid is added at the time of draining),}
\]

\[
\frac{d}{dt}(M_{tot}) = -W_2
\]

\[
M_{tot} = \text{Total mass of liquid in the tank } \left( \frac{1}{3} \pi r^2 h \rho \right).
\]

The mechanical energy balance equation (Bernoulli theorem) between station-1 and station-2 can be written as

\[
\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gZ_2 + \frac{4fL}{2d}V_2^2
\]

Since the inlet and outlet are open to atmosphere and the liquid drains slowly, \( P_1 = P_2 \).
Fig.1 Conical tank along with the exit pipe assembly

\[ V_1 = 0 \] (Since the liquid drains very slowly and this assumption is known as pseudo steady state assumption)

At any height \( h \) and \( L \) noting that \( Z_1 = Z_2 + h + L \)

Substituting these values in Eqn. (3)

\[ g(h + L) = \frac{V_2^2}{2} + \frac{4fLV_2^2}{2d} \]

... .......................... (4)

Assuming constant friction factor, the above equation can be written as

\[ g(h + L) = \frac{V_2^2}{2} \times (1 + 4fL/d) \]

............................... (5)

\[ V_2^2 = \frac{2 \times g \times (h + L)}{1 + 4f(L/d)} \]

............................... (6)

\[ V_2 = \sqrt{\frac{2 \times g \times (h + L)}{1 + 4f(L/d)}} \]

......................... (7)

\[ W_2 = V_2 \rho A_p \]

......................... (8)

Where \( A_p = (\pi/4)d^2 \)

Substituting \( V_2 \) from Eqn.(7) in eqn.(8)

\[ W_2 = \frac{2 \times g \times (h + L)}{\sqrt{1 + 4f(L/d)}} \times \rho \times \left(\frac{\pi}{4}\right) \times d^2 \]

............................... (8)
Substituting the value of \( W_2 \) and \( m_{\text{tot}} \) in Eqn. (2)

Applying mass balance equation

\[
\frac{d}{dt}\left(\frac{1}{3}\pi r^2 h \rho\right) = -\rho \sqrt{\frac{2g(h+L)}{1+4f(L/d)}} \times \frac{\pi}{4} \times d^2
\]

.......... (9)

Here \( \rho \) is constant,

From Fig.1

\[
r = \frac{R}{H}
\]

\[
r = \frac{h \times R}{H}
\]

.......... (10)

Substituting “r” value in the above equation (9)

\[
\frac{d}{dt}\left(\frac{1}{3}\left(\frac{h \times R}{H}\right)^2 h\right) = -\frac{\pi}{4} \times \frac{d^2}{4}
\]

.......... (11)

\[
h^2 \frac{dh}{\sqrt{h+L}} = -\frac{d^2}{4R^2} \times H^2 \sqrt{\frac{2g}{1+4f(L/d)}} dt
\]

.......... (12)

Integrating on both sides

\[
\int_{\frac{h}{H}}^{h'} \frac{h^2}{\sqrt{h+L}} \, dh = -\int_{0}^{t} \frac{d^2}{4R^2} \times H^2 \sqrt{\frac{2g}{1+4f(L/d)}} \, dt
\]

.......... (13)

At \( t=0, \ h=H \) and \( t=t, \ h=H' \)

\[
\int_{\frac{h}{H}}^{h'} \frac{h^2}{\sqrt{h+L}} \, dh = \int_{0}^{t} \frac{d^2}{4R^2} \times H^2 \sqrt{\frac{2g}{1+4f(L/d)}} \, dt
\]

.......... (14)

By applying integration for L.H.S of equation (14),

\( h+L = X \)

\( h = X-L \)

\( DH = dX \)

Substituting these values in the above equation (14)

\[
\int_{\frac{h}{H}}^{h'} \frac{h^2}{\sqrt{h+L}} \, dh = \int_{\frac{h}{H}}^{H'} \frac{(X-L)^2}{\sqrt{X-L+L}} \, dX
\]
The above equation can further be written in dimensionless form as

\[
t_{\text{eff}} = \left[0.4(H + L)^{2/3} - (H' + L)^{2/3}\right] + 2L^2 \left(\sqrt{H + L} - \sqrt{H' + L}\right) - 1.33L \left[(H + L)^{3/2} - (H' + L)^{3/2}\right]
\]

\[
= \frac{4R^2}{H^2 d^2} \left[\frac{(1 + 4f(L/D))}{2g}\right] \frac{(L/H)^2}{d^2} \left[0.4 \left(1 + \frac{H'}{L}\right)^{2/3} - 2 \left(1 + \frac{H'}{L}\right)^{3/2} - 1.33 \left(1 + \frac{H'}{L}\right)^{3/2} - \left(1 + \frac{H'}{L}\right)^{3/2}\right]
\]

....... (15)

III. EXPERIMENTAL PROCEDURES

A. Description of the equipment

The equipment used for experimentation consisted of known diameters (0.52, 0.50 and 0.45m) of the stainless steel conical tanks. A mild steel pipe of 6.8 x 10^-3 m diameter (d) was fitted at the centre of the bottom of the tank, served as an exit pipe. A Gate valve (GV) was provided at the bottom most point of the exit pipe, served as control valve for draining of liquid from the tank. A transparent plastic tube (LI) was provided to the tank served as level indicator during the draining operation of the tank, served as an exit pipe. A Gate valve (GV) was provided at the bottom most point of the exit pipe, served as control valve for draining of liquid from the tank. A transparent plastic tube (LI) was provided to the tank served as level indicator during the draining operation of the tank, served as an exit pipe. A Gate valve (GV) was provided at the bottom most point of the exit pipe, served as control valve for draining of liquid from the tank.

B. Preparation of polymer solution

Polyacrylamide (PAM) used in the present study was obtained from Otto Chemmi-Mumbai. The manufacturer quoted its molecular weight as 5,000,000. The stock solution of PAM was prepared by dissolving 1g of Polyacrylamide (PAM) in 1000ml of water. The solution was stirred for 4 hours and then allowed to hydrate for 24 hours. The solution without any non-homogeneity was diluted suitably to prepare 40, 30, 20 and 10ppm solutions. The pre-mixed solutions were added to the conical tank to obtain the efflux time. The efflux time was measured using a stopwatch with 1 second accuracy.

C. Efflux time measurement for with and without polymer solutions

Gate valve (GV) was closed and the tank along with the exit pipe was filled with water/polymer solution as the case may be up to the mark and allowed to stabilize. The stop watch was started immediately after the opening of the bottom Gate valve. The fall in level was read from the level indicator. The time was recorded for a known drop in the liquid level. The measurements were continued till the water level reaches to a desired value i.e. 9cm just above the tank bottom. The experimental efflux time was designated as t_{act}. The experiments were repeated to check the consistency and reproducibility of data. The procedure was repeated for all the cases of \( A_t / A_p \) ratios.

IV. RESULTS AND DISCUSSION

A. General observations

The objective of the present study is to reduce the efflux time during the draining of a liquid through a gravity driven system both in the absence and presence of PAM additives. This is achieved by the following methods outlined below and the experimental data on efflux time were obtained for the following cases.

a) Draining a liquid through a single exit pipe system in the absence of polymer additives.

b) Draining a liquid solution through a single exit pipe system in the presence of Polyacrylamide (PAM) polymer additives.

The methods of draining operations are shown in Figs.2-4 for the case of L =1m and H=0.43m. The tank is drained to a minimum depth of 0.09 m above the bottom in all the cases as complete draining is not practically
carried out and also to ensure that the flow is essentially due to gravity and the data are shown as the variation of height of the liquid in the tank with time.

Fig. 2 Draining pattern for with and without polymer concentrations for single exit pipe 
( H=0.43m, D=0.52m, d=0.006m, L=1m)

Fig. 3 Draining pattern for with and without polymer concentrations for single exit pipe 
( H=0.41m, D=0.50m, d=0.006m, L=1m)
B. Comparison of efflux time data

The equations developed to obtain the efflux time are derived based on the gravitational component of the force acting on the bulk fluid. In a closed system, when the liquid is under pressure and is far greater than the gravitational force, the quasi-steady state equations hold true and the unsteady state flow terms can be neglected. But when only gravity is acting on the liquid, the unsteady state flow equations must be considered. In the present case, gravity is the only force acting and hence even though pseudo steady state conditions are assumed; only unsteady state conditions prevail. Since the analysis is based on macroscopic balances, fluid motion within the conical tank is also neglected. As the experimental values account not only for the above, but also the contraction coefficient and roughness of the pipe, deviations between efflux time values based on Eqn. 16 and the experimentally measured values are hence expected. The following friction factor term for the range of Reynolds number $3000$ to $3 \times 10^6$ can be used to obtain the efflux time from Eqn.16.

$$f = 0.0014 + \frac{0.125}{\text{Re}^{0.32}}$$

... (17)

Where $\text{Re} = \frac{DV_{\text{exp}}\rho}{\mu}$

$V_{\text{exp}}$ is obtained using the experimentally measured data as

$$V_{\text{exp}} = \left[ \frac{\pi}{3} R^2 \times H - \frac{\pi}{3} \times r^2 \times H \right] \left[ \frac{\pi}{4} d^2 \times t_{\text{act}} \right]$$

...... (18)

The values of $f$ and $V_{\text{exp}}$ are substituted in Eqn.1 to arrive at $t_{eq}$. Experimental values of $t_{\text{act}}$ are compared with $t_{eq}$ to check the validity of mathematical equation for efflux time. The efflux time data on both $t_{eq}$ and $t_{\text{act}}$ in seconds are plotted against

Fig. 4 Draining pattern for with and without polymer concentrations for single exit pipe

(H =0.41m, D=0.45m, d=0.006m, L=1m)
\[ C = \left[ 0.4 \left( \frac{H}{L} \right)^{25} - \left( \frac{H}{L} \right) \right] + 2 \left( 1 + \frac{H}{L} \right) - 1.33 \left( \left( \frac{H}{L} \right)^{3/2} - \left( \frac{H}{L} \right) \right) \] for the cases, \( D=0.52 \text{m}, \)

\( D=0.50 \text{m} \) and \( D=0.45 \text{m} \) diameter tanks with constant exit pipe length of 1m and diameters of the exit pipes 0.006m are shown in Figs.5-8.

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**Fig.5** Comparison of efflux time before fine-tuning (\( D=0.52 \text{m}, \ d=0.006 \text{m}, \ L=1\text{m} \))

**Fig.6** Comparison of efflux time before fine-tuning (\( D=0.45 \text{m}, \ d=0.006 \text{m}, \ L=1\text{m} \))
The figures show deviation of $t_{eq}$ calculated based on friction factor equation (17) from measured efflux time $t_{act}$. In view of the large deviations between theoretical and experimental values, an iterative technique is used to obtain following fine tuned friction factor equation

$$f = 0.0014 + \frac{0.125}{Re^{0.17}} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (19)$$

**Fig.7** Comparison of efflux time after fine-tuning ($D=0.52m$, $d=0.006m$, $L=1m$)

C. Effect of initial height of liquid in the tank

The tank was drained to a minimum depth of 0.09m above the bottom of the conical tank in all the cases as complete draining is not practically carried out and also to ensure that the flow is essentially due to gravity and the data are shown as the variation of height of the liquid in the tank with time. Variation of efflux time for
different initial heights of liquid in the tank was carried out for zero ppm solution (D=0.52m and L=1) is shown in Fig.9. Higher efflux times are obtained as the initial height of liquid in the tank is increased.

![Graph showing efflux time vs initial height for D=0.52m and L=1m, d=0.006m](image)

**D. Effect of \( \frac{A_t}{A_p} \) on efflux time**

As shown in the Fig.10, for different exit pipe lengths (L=1, 0.75, 0.50 and 0.25m), the effect of At/ Ap ratio was plotted with efflux time, for different initial heights of the liquid levels in the tank and it was observed as \( \frac{A_t}{A_p} \) ratio decreases, that is as the initial height of the liquid level in the tank decreases efflux time also decreases.

![Graph showing efflux time vs At/ Ap ratio for different exit pipe lengths](image)
E. Study of tank draining pattern (with and without polymer additives)

As shown in Fig. 11 for different ppm polymer solutions, variation of efflux time was plotted with different initial heights of the liquid levels in the tank and the following observations were made from the analysis of the data (D=0.52m, d=0.006 L=1m),

% Drag reduction in the present case in terms of efflux time is defined as:

\[ \% \text{ DR} = \left( \frac{t_{\text{effuntreated}} - t_{\text{efftreated}}}{t_{\text{effuntreated}}} \right) \times 100 \]

\( t_{\text{effuntreated}} \) and \( t_{\text{efftreated}} \) refer to efflux time in the absence and presence of polymer.

- Lower draining time is observed in the presence of polymer solution (PAM).
- As the diameter of the exit pipe increases the time taken to drain the solution decreases.
- The presence of polymer solution is found to affect the draining time certainly by a minimum of 18% on an average.
- The presence of polymer solution is found to affect the draining time certainly by a maximum of 26% on an average.
- The graph is curved due to tapered cross section of the conical tank.

Fig. 11 Draining Pattern for Different Polymer Concentrations for Single Exit Pipe

(D=0.52m, d=0.006m, L=1m)
Draining of tank contents was found to be relatively faster with PAM solutions up to 30ppm and after that slightly slow down. Polymer addition to the vessel show significant reduction in % drag reduction. i.e., Draining of tank contents was found to be relatively faster when polymer solutions were added.

For PAM concentration of 30ppm, draining of tank solution with minimum efflux time was observed.

The efflux time was increased with an increase in length of the exit pipe.

The efflux time was decreased with an increase in exit pipe diameter.

The efflux time was increased with an increase in tank diameter.

The efflux time was decreased with decrease in height of liquid in the tank.

For presence of polymer solution, % of drag reduction was found certainly by a minimum of 18% on an average.

For presence of polymer solution, % of drag reduction was found certainly by a maximum of 26% on an average.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( A_p )</td>
<td>Area of exit pipe, ( m^2 )</td>
</tr>
<tr>
<td>( A_t )</td>
<td>Area of tank, ( m^2 )</td>
</tr>
<tr>
<td>( D )</td>
<td>Diameter of tank, m</td>
</tr>
<tr>
<td>( d )</td>
<td>Diameter of exit pipe, m</td>
</tr>
<tr>
<td>( f )</td>
<td>Friction factor, dimensionless</td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration due to gravity, ( m/sec^2 )</td>
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<tr>
<td>( c )</td>
<td>Efflux time proportional constant</td>
</tr>
<tr>
<td>( GV )</td>
<td>Gate Valve</td>
</tr>
<tr>
<td>( H )</td>
<td>initial height of liquid in the tank at ( t=0 ), m</td>
</tr>
<tr>
<td>( H^f )</td>
<td>Final height of liquid in the tank on draining, m</td>
</tr>
<tr>
<td>( h )</td>
<td>Height of liquid in the tank at any time, m</td>
</tr>
<tr>
<td>( h_{fs} )</td>
<td>skin friction losses, j/kg</td>
</tr>
<tr>
<td>( L )</td>
<td>Length of the exit pipe, m</td>
</tr>
<tr>
<td>( LI )</td>
<td>Level indicator</td>
</tr>
<tr>
<td>( m_{tot} )</td>
<td>total mass of liquid in the tank, kg</td>
</tr>
<tr>
<td>( Re )</td>
<td>Reynolds number, Dimensionless</td>
</tr>
<tr>
<td>( t_{eq} )</td>
<td>Efflux time, sec</td>
</tr>
<tr>
<td>( t_{act} )</td>
<td>Actual efflux time, sec</td>
</tr>
<tr>
<td>( t_{eq}^* )</td>
<td>Efflux time based on fine-tuned friction factor</td>
</tr>
</tbody>
</table>

Equation

\[ P_1 & P_2 = \text{Pressures at station 1 and station 2 respectively, N/m}^2 \]

\[ V_1 & V_2 = \text{Velocities at station 1 and 2 respectively, m/sec} \]
\[ V_{2exp} = \text{Experimental average velocity, m/sec} \]

\[ W_1 \text{ & } W_2 = \text{Mass flow rate at 1 and 2 respectively, kg/sec} \]

\[ Z_1 \text{ & } Z_2 = \text{Elevation at station 1 and 2 respectively, m} \]

\[ \rho = \text{Density of liquid, kg/m}^3 \]

\[ \mu = \text{Viscosity of liquid, kg/m.sec} \]

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