Abstract—The graph $G$ is Hamiltonian laceable [2] if there exists a Hamiltonian path between every pair of distinct vertices in it at an odd distance. $G$ is Hamiltonian-t-laceable ($t^{*}$-laceable) if there exists a Hamiltonian path in $G$ between every pair (at least one pair) of vertices $u$ and $v$ in $G$ with the property $d(u,v) = t$. In this paper, we discuss the Hamiltonian laceability properties of the graph $G * v$, where $G$ is the Star graph $G = K_{1,n}$, $(n \geq 3)$. We also explore the Hamiltonian Laceability properties of the subdivision graph $G^{+}$.

Index Terms—Hamiltonian path, Hamiltonian laceability, Hamiltonian-t-laceable path, i-Hamiltonian laceability.

I. INTRODUCTION

Let $G$ be a finite, simple connected undirected graph. Let $u$ and $v$ be two vertices in $G$. The order of $G$ denoted by $O(G)$ is the cardinality of the vertices of $G$. The distance between $u$ and $v$ denoted by $d(u,v)$ is the length of a shortest $u$-$v$ path in $G$. $G$ is Hamiltonian Laceable if there exists a Hamiltonian path between every pair of distinct vertices in it at an odd distance. $G$ is Hamiltonian-t-laceable if there exists a Hamiltonian path between every pair of vertices $u$ and $v$ in $G$ with the property $d(u,v) = t$ and Hamiltonian-t*-laceable [2] if there exists a Hamiltonian path between at least one such pair with the property $d(u,v) = t$, where $t$ is a positive integer such that $1 \leq t \leq diam(G)$. Hamiltonian laceability in the brick product of cycles was explored by B. Alspach, C.C. Chen and Kevin McAveney in [1] where the authors proved the laceability in the brick product of odd cycles. Hamiltonian-–t-laceability in the brick product of even cycles was studied by Leena. N. Shenoy and R. Murali in [2]. In [3], Girisha. A and R. Murali have studied Hamiltonian-t*-laceability of 4-regular graphs. In this paper we study the Hamiltonian-t*-laceability properties of the graph extended star graph $G * v$ and the subdivision graph $G^{+}$.

Definition 1.1: Let $G = K_{1,n}$ be the star graph and $v \in V(K_{1,n})$. The graph $G * v$ is obtained from $G$ by replacing the vertex $v$ by a cycle of length $n$ and joining the vertices of the cycle to the former neighbors of $v$ as shown in Fig.1.

Fig. 1: The Graphs $G$ and $G * v$
Definition 1.2: Let $G$ be a complete graph. The subdivision graph obtained by inserting a vertex of degree two into any one edge of $G$ and we denote it by $G^+$.

When the inserted vertex in a subdivision of $G$ is specified, say $u$, we denote by $G(u)$ a graph with $V(G(u)) = V(G) \cup \{u\}$ and $E(G(u)) = (E(G) - xy) \cup \{ xu, xy \}$ where $xy \in E(G)$.

Definition 1.3: Let $G$ be a connected graph of order $n$ and let $h_p(G)$ be the length of a Hamiltonian path [4] between any two distinct vertices in $G$. A Hamiltonian path in $G$ is called a 0-Hamiltonian path if $h_p(G) = n - 1$ and a 1-Hamiltonian path if $h_p(G) = n$.

Definition 1.4: Let $i$ be a non-negative integer. A connected graph $G$ of order $n$ is called $i$-Hamiltonian-$t$-laceable if there exists in $G$, a $i$-Hamiltonian path between every pair of distinct vertices $u$ and $v$ with the property $d(u, v) = t$, $1 \leq t \leq \text{diam} G$.

Definition 1.5: A connected graph $G$ of order $n$ is called $i$-Hamiltonian-$t^*$-laceable if there exists in $G$, an $i$-Hamiltonian path [4] between at least one pair of distinct vertices $u$ and $v$ with the property $d(u, v) = t$, $1 \leq t \leq \text{diam} G$.

Definition 1.6: Let $G = K_{1,n}$, $n \geq 3$, be the star graph of order $n$. Then the extended star graph $K_{1,n,n}$ is obtained by inserting a star graph of order $n - 1$ to each pendant vertex of $K_{1,n}$.

II. RESULTS

Theorem 2.1: The graph $G = K_{1,n} \ast v$, $n \geq 3$ is $i$-Hamiltonian-$1^*$-laceable for $i = n$.

Proof: Let us denote the vertices of $K_{1,n} \ast v$ by $a_1,a_2,a_3,a_4,a_5,\ldots,a_n$ and $b_1,b_2,b_3,b_4,b_5,\ldots,b_n$. Here we need to establish the following case to show that $G$ is $i$-Hamiltonian-$1^*$-laceable.
In $G$, $d(b_1, a_i) = 1$ and the path

$$P : (b_1, a_2) \cup (a_2, b_2) \cup (b_2, a_3) \cup (a_3, b_3) \cup (b_3, a_4) \cup (a_4, b_4) \cup (b_4, a_5) \cup (a_5, b_5) \cup (b_5, a_6) \cup (a_6, b_6) \cup \cdots \cup (a_{n-1}, b_{n-1}) \cup (b_{n-1}, a_n) \cup (a_n, b_n) \cup (b_n, a_1)$$

is a Hamiltonian path from $b_1$ to $a_1$ in $G$.

Hence the proof.

**Theorem 2.2:** The $G = k_{1,n} * v$, $n \geq 3$ is i-Hamiltonian-2'-laceable for $i = n - 1$.

**Proof:** Let us denote the vertices of $K_{1,n} * v$ by $a_1, a_2, a_3, a_4, a_5, \ldots, a_n$ and $b_1, b_2, b_3, b_4, b_5, \ldots, b_n$. Here we need to establish the following case to show that $G$ is i-Hamiltonian-2'-laceable. In $G$, $d(b_1, a_2) = 2$ and the path

$$P : (a_2, b_2) \cup (b_2, a_3) \cup (a_3, b_3) \cup (b_3, a_4) \cup (a_4, b_4) \cup (b_4, a_5) \cup (a_5, b_5) \cup (b_5, a_6) \cup (a_6, b_6) \cup \cdots \cup (a_{n-1}, b_{n-1}) \cup (b_{n-1}, a_n) \cup (a_n, b_n) \cup (b_n, a_1)$$

is a Hamiltonian path from $b_1$ to $a_2$ in $G$.

Hence the proof.

**Theorem 2.3:** Let $G$ be the complete graph of order $n$ ($n \geq 3$). Then $G^+$ is 1-Hamiltonian-2'-laceable.

**Proof:** Let $G = k_n$ ($n \geq 3$) be the complete graph and $G^+$ be the subdivision graph obtained by inserting a vertex $u$ of degree two into any edge of $G$ with the end vertices $x$ and $y$ such that $d(x, y) = 2$. $G^+$ has $n + 1$ vertices and $^cC_2 + 1$ edges.

Let $u, x, a_1, a_2, a_3, a_4, a_5, \ldots, a_{n-2}, y$ be the vertices of $G^+$.

Then the path

$$P : (x, a_1) \cup (a_1, a_2) \cup (a_2, a_3) \cup (a_3, a_4) \cup (a_4, a_5) \cup (a_5, a_6) \cup \cdots \cup (a_{n-3}, a_{n-2}) \cup (a_{n-2}, u) \cup (u, y)$$

is a Hamiltonian - 2 *-laceable path from $x$ to $y$.

Hence the proof.

**Theorem 2.4:** The graph $k_{1,n}$ is i-Hamiltonian-1'-laceable for $i = O(k_{1,n}) - 3$.

**Proof:** Let $G = k_{1,n}$ be a star graph of order $n$ and $G_1 = k_{1,n}$ be an extended star graph with vertices $b_1, b_2, b_3, b_4, b_5, \ldots, b_{n-3}, b_{n-2} - b_n, b_{n-1}$ and $a_1, a_2, a_3, a_4, a_5, \ldots, a_n$ and a parent vertex $v$.

![Fig. 3: The graph $k_{1,n}$ and $k_{1,n}$](image-url)
In $G_1$, $d(a_1, v) = 1$ and the path

$$P: (a_1, b_1) \cup (b_1, b_2) \cup (b_2, b_3) \cup \ldots \ldots \cup (b_{n-1}, a_2) \cup (a_2, b_n) \cup (b_n, b_{n+1}) \cup (b_{n+1}, b_{n+2}) \cup (b_{n+2}, b_{n+3}) \cup (b_{n+3}, b_{n+4}) \cup \ldots \ldots \cup (b_{(3n-4)}, b_{(2n-3)}) \cup (b_{(2n-3)}, a_4) \cup (a_4, b_{(2n-2)}) \cup (b_{(2n-2)}, b_{(2n-1)}) \cup (b_{(2n-1)}, b_{2n}) \cup (b_{2n}, b_{2n+1}) \cup \ldots \ldots \cup \cup (b_{3n+2}, b_{3n+3}) \cup (b_{3n+3}, a_5) \cup (a_5, b_{3n+4}) \cup (b_{3n+4}, b_{3n+5}) \cup \ldots \ldots \cup (b_{4n+1}, b_{4n+2}) \cup \ldots \ldots \cup \cup (a_n, b_{(n-2)n+2}) \cup (b_{(n-2)n+2}, b_{(n-2)n+3}) \cup (b_{(n-2)n+3}, b_{(n-2)n+4}) \cup \ldots \ldots \cup (b_{(n-2)n+n}, v)$$

is a $i- \text{hamiltonian} - 1^* - \text{laceable path from} a_1 \text{ to } v$ with $i = O(k_{1,n,n}) - 3$.

Hence the proof

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