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TE TM Electromagnetic Fields and Potentials

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***Abstract :** TE, TM harmonic Fields are obtained from Potentials in cartesian and cylindrical coordinadtes. In both cases and, in presence of charges and currents, these potentials for the Lorentz gauge, are solutions of 2D in homogeneous Helmholtz equations, solved with the help of Green functions. Solutions are made explicit for currents generated by moving point charges.*

Keywords: **TE, TM Fields, potentials, Green function.**

I. INTRODUCTION

Scalar and vector potentials are a powerful tool to analyze many different problems in elec-tromagnetism. For instance, in presence of charges and currents ρ, J, in a homogeneous iso-tropic medium with permittivity ε and permeability μ, the electric and magnetic fields **E**, **H** are solutions of the inhomogeneous wave equations

$$\Delta \mathbf{E} - \varepsilon\mu/c^2\, \partial_t^2 \mathbf{E} = \mu \partial_t \mathbf{J} + \nabla(\rho/\varepsilon) \quad , \quad \Delta \mathbf{H} - \varepsilon\mu/c^2\, \partial_t^2 \mathbf{H} = -\nabla \wedge \mathbf{J} \qquad (0)$$

while in the Lorentz gauge [1,2], the potentials **A**,ϕ with

$$\mu \mathbf{H} = \nabla \wedge \mathbf{A} \quad , \quad \mathbf{E} = -\nabla\phi - 1/c\, \partial_t \mathbf{A} \qquad (1)$$

satisfy the more sympathetic wave equations

$$\Delta \mathbf{A} - \varepsilon\mu/c^2\, \partial_t^2 \mathbf{A} = -\mu \mathbf{J} \quad (2a) \quad , \quad \Delta\phi - \varepsilon\mu/c^2\, \partial_t^2 \phi = -\rho/\varepsilon \qquad (2b)$$

which may be solved with the help of Green functions.
Note that A,ϕ are subject to the constraint

$$\nabla . \mathbf{A} + \varepsilon\mu/c\, \partial_t \phi = 0 \qquad (2c)$$

fields and potentials being associated respectively with force and energy.
Now, for TE, TM harmonic fields, also discussed succinctly in [1], we are concerned with 2D fields and potentials so that Eqs.(2a,b) become 2D inhomogeneous Helmholtz equations. It is imposed that the currents in cylindrical coordinates are such that these equations become se-parable.There solutions in terms of Green functions are discussed in cartesian and cylindrical coordinates with an application to currents generated by moving point charges.

II. FIELDS AND POTENTIALS IN CARTESIAN COORDINATES (x,y,z)

In cartesian coordinates Eqs.(1) become

$$\mu H_x = \partial_y A_z - \partial_z A_y \quad , \quad E_x = -\partial_x \phi - 1/c\, \partial_t A_x$$
$$\mu H_y = \partial_z A_x - \partial_x A_z \quad , \quad E_y = -\partial_y \phi - 1/c\, \partial_t A_y$$
$$\mu H_z = \partial_x A_y - \partial_y A_x \quad (3a) \quad , \quad E_z = -\partial_z \phi - 1/c \partial_t A_z \quad (3b)$$

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implying **∇. H** = **∇. E** = 0. From now on, we assume harmonic fields and potentials with exp(−iωt) implicit and in (3b) ∂_t changed into −iω.

A. TE FIELDS

We consider the TE fields whose components are only function of x,z

$$E_y(x,z) \quad , \quad H_x(x,z) \quad , \quad H_z(x,z) \qquad (4a)$$

with potentials, charges and currents

$$A_x = A_z = 0\, ,\ A_y(x,z)\, , \quad \phi = 0 \quad ; \quad J_x = J_z = 0\ ,\ J_y(x,z) \quad , \quad \rho = 0 \qquad (4b)$$

Then, we get from (3a,b)

$$\mu H_x = -\partial_z A_y \quad , \quad \mu H_z = \partial_x A_y \quad , \quad E_y = -1/c\ \partial_t A_y \qquad (5a)$$

while according to (2a), the inhomogeneous Helmholtz equation satisfied by A_y is

$$(\delta_x^2 + \partial_z^2 + \omega^2\varepsilon\mu/c^2)\, A_y = -\mu J_y \qquad (5b)$$

To solve (5b), we look for the Green function G solution of the equation

$$(\delta_x^2 + \partial_z^2 + \omega^2\varepsilon\mu/c^2)\, G(x,z, ; x',z') = -4\pi\, \delta(x-x')\, \delta(z-z') \qquad (6)$$

which is [1-3] is an infinite medium

$$G(x,z, ; x',z') = i\pi\, H_0^{(1)}(kr), \qquad (6a)$$

with

$$k = \omega(\varepsilon\mu)^{1/2}/c \ , \quad r = [(x-x')^2 + (z-z')^2]^{1/2} \qquad (6b)$$

while $H_0^{(1)}(kr)$ is the Hankel function of the first kind of order zero.

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Then, the solution of the Helmholtz equation (5a) is

$$A_y(x,z) = i\mu/4 \iint dx'\, dz'\, J_y(x',z')\, H_0^{(1)}(kr) \qquad (7)$$

Using (7) we explicit the TE wave (4a) for a line current

$$J_y = j_y\, \delta(x - x_1 - \lambda x_2)\, \delta(z - z_1 - \lambda z_2) \qquad (8)$$

j_y is a constant amplitude with dimension $L^{-1}J$, λ an arbitrary real parameter and δ the delta function. Substituting (8) into (7) gives

$$A_y(x,z) = i\mu j_y /4\, H_0^{(1)}(kr_{1,2}) \qquad (9)$$

where

$$r_{1,2} = [(x - x_1 - \lambda x_2)^2 + (z - z_1 - \lambda z_2)^2]^{1/2} \qquad (9a)$$

Then, according to (5a) and to the relations

$$\partial_z H_0^{(1)}(kr) = -k\, H_1^{(1)}(kr)\, \partial_z r \quad ; \quad \partial_x H_0^{(1)}(kr) = -k\, H_1^{(1)}(kr)\, \partial_x r \qquad (10)$$

the components of the TE wave have the expressions

$$E_y(x,z) = -\, \mu\omega j_y/4c\, H_0^{(1)}(kr_{1,2})$$
$$H_x(x,z) = ikj_y/4\, H_1^{(1)}(kr_{1,2})\, \partial_z r_{1,2}$$
$$H_x xx,z) = -ikj_y/4\, H_1^{(1)}(kr_{1,2})\, \partial_x r_{1,2} \qquad (11)$$

Suppose now a current generated by a point charge moving with the velocity $v_y(x_0,z_0)$

$$J_y(x,z) = ev_y(x_0,z_0)\, \delta(x - x_0)\, \delta\, (z - z_0) \qquad (12)$$

where the charge density e has a dimension consistent with the electromagnetic field dimen-sions and from now on we write $\underline{v}_y(x_0,z_0)$ for $ev_y(x_0,z_0)$.

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Substituting (12) into (7) gives

$$A_y(x,z) = i\mu\underline{v}_y /4\, H_0^{(1)}(kr^{\dagger}) \quad , \quad r^{\dagger} = [(x - x_0)^2 + (z - z_0)^2]^{1/2} \qquad (13)$$

The comparison of (9) and (13) shows that changing in (11) j_y , $r_{1,2}$ into $\underline{v}_y$, $r^{\dagger}$ gives the com-ponents of the TE field generated by the current (12) in which x_0, z_0, is the position of the point charge.

Remark. 1 : These results may be generalized to situations where the Green function satisfies on the z = 0 plane either the Dirichlet boundary condition A(x,0) = f(x) or the Neumann boundary condition $[\partial_z A(x,z)]_{z=0} = f(x)$. In the first case the solution of Eq.(5b) is [4]

$$A_y(x,z) = \mu \int_{-\infty}^{\infty} dx'\, f(x')\, [\partial_z G_d(x,z\, ;\, x',z')]_{z=0} - \mu \int_0^{\infty}\!\!\int_{-\infty}^{\infty} dx'\, dz\, j_y(x,z')\, G_d(x,z\, ;\, x',z') \qquad (14)$$

in which

$$G_d(x,z\, ;\, x',z') = i\pi/4\, [H_0^{(1)}(kr_1) - H_0^{(1)}(kr_2)] \qquad (15)$$

with

$$r_1 = [(x - x')^2 + (z - z')^2]^{1/2} \quad , \quad r_2 = [(x - x')^2 + (z + z')^2]^{1/2} \qquad (15a)$$

while for the Neumann boundary condition G_d is changed into G_n

$$G_n(x,z\, ;\, x',z') = i\pi/4\, [H_0^{(1)}(kr_1) + H_0^{(1)}(kr_2)] \qquad (16)$$

B.TM FIELD

We consider the TM field with components

$$H_y(x,z) \quad , \quad E_x(x,z) \quad , \quad E_z(x,z) \qquad (17a)$$

while the potentials, charges and currents are

$$A_y = 0,\ A_x(x,z)\, ,\ A_z(x,z)\, ,\ \phi(x,z) \quad ; \quad J_y = 0 \quad , \quad J_x(x,z) \quad , \quad J_z(x,z), \quad \rho(x,z) \qquad (17b)$$

and, we get from (3a,b)

$$\mu H_y = \partial_z A_x - \partial_x A_z \quad , \quad E_x = -\partial_x\phi + i\omega/cA_x \quad , \quad E_z = -\partial_z\phi + i\omega/cA_z \qquad (18)$$

We first start with the components A_x, A_z of the potential satisfying the inhomogeneous Hel-mholtz equations ($k^2 = \omega^2\varepsilon\mu/c^2$)

$$(\partial_x^{\,2} + \partial_z^{\,2} + k^2)\, A_{x,z} = -\mu J_{x,z} \qquad (19)$$

so that changing y into x,z successively, we get fom (7) with $r = [(x-x')^2 + (z-z')^2]^{1/2}$

$$A_{x,z}(x,z) = i\mu/4 \iint dx'\, dz'\, J_{x,z}(x',z')\, H_0^{(1)}(kr) \qquad (20)$$

Then, for a line current J, the components of the TM field have the expressions (8) with J_y changed into $J_{x,z}$ so that according to (9) with $r_{1,2}$ given by (9a)

$$A_{x,z}(x,z) = i\mu\, J_{x,z} /4\, H_0^{(1)}(kr_{1,2}) \qquad (21)$$

Substituting (21) into (18) and using (10) gives the components of the TM field

$$E_x(x,z) = -\partial_x\phi\ -\ \mu J_x/4c\, H_0^{(1)}(kr_{1,2})$$
$$E_z(x,z) = -\partial_z\phi - \mu\omega J_z/4c\, H_0^{(1)}(kr_{1,2})$$
$$H_y(x,z) = ik\pi/4\, H_1^{(1)}\, (kr_{1,2})\, (J_x\partial_z r_{1,2} - J_z\partial_x r_{1,2}) \qquad (22)$$

in which ϕ has still to be determined while $\partial_r H_0^{(1)}(kr_{1,2}) = -\, k\, H_1^{(1)}(kr_{1,2})$.

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Now, for a moving point charge, whose velocity has the components $\underline{v}_x(x_0,z_0)$, $\underline{v}_z(x_0,z_0)$, the

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components $J_{x,z}$ of the current are

$$J_{x,z}(x,z) = \underline{v}_{x,z}(x_0,z_0)\,\delta(x-x_0)\,\delta(z-z_0) \qquad (23)$$

so that according to (13) with $k = \omega\sqrt{\varepsilon\mu}/c$

$$A_{x,z}(x,z) = i\mu\underline{v}_{x,z}/4\, H_0^{(1)}(kr^{\dagger}) \qquad (24)$$

and substituting (24) into (18), we get for the components of the TM field

$$E_x(x,z) = -\partial_x\phi - \mu\omega\,\underline{v}_x/4c\, H_0^{(1)}(kr_{1,2})$$
$$E_z(x,z) = -\partial_z\phi - \mu\omega\,\underline{v}_z/4c\, H_0^{(1)}(kr_{1,2})$$
$$H_y(x,z) = ik/4\, H_1^{(1)}(kr^{\dagger})\,(\underline{v}_x\partial_z r^{\dagger} - \underline{v}_z\partial_x r^{\dagger}) \qquad (25)$$

These results may be generalized when the potentials satisfy on the z = 0 plane either the Diri-chlet or the Neumann boundary condition but this generalization is a bit more intricate than for TE waves according that one or both components of the 2D vector **A** satisfy the bounda-ry condition.

We now have to discuss the scalar potential $\phi(x,z)$ solution according to (2b) of the 2D Hel-mholtz equation($k^2 = \omega^2\varepsilon\mu/c^2$)

$$(\partial_x^2 + \partial_z^2 + k^2)\phi = -\rho/\varepsilon \qquad (26)$$

so that, still using the Green function $i\pi\, H_0^{(1)}(kr)$, $\qquad r = [(x-x')^2 + (z-z')^2]^{1/2}$

$$\phi(x,z) = i/4\varepsilon \iint dx'\, dz'\, \rho(x',z')\, H_0^{(1)}(kr) \qquad (27)$$

The constraint (2c) between charge and current has still to be satisfied which implies

$$i\omega\varepsilon\mu/c\ \phi = -(\partial_x A_x + \partial_z A_z) \qquad (28)$$

where according to (20)

$$\partial_x A_x + \partial_z A_z = -i\mu k/4 \iint dx'dz'\, H_1^{(1)}(kr)\,[J_x(x',z')\,\partial_x r + J_z(x',z')\,\partial_z r] \qquad (29)$$

Substituting (27) and (29 into (28) gives

$$\omega/ck \iint dx'dz'\, \rho(x',z')\, H_0^{(1)}(kr) = \iint dx'dz'\, H_1^{(1)}(kr)\,[J_x(x',z')\,\partial_x r + J_z(x',z')\,\partial_z r] \qquad (30)$$

$$\partial_x r = (x-x')/r \quad , \quad \partial_z r = (z-z')/r \qquad (30a)$$

Suppose now the currents generated by a moving point charge, then according to (23)

$$J_{x,z}(x',z') = \underline{v}_{x,z}(x_0,z_0)\,\delta(x'-x_0)\,\delta(z'-z_0) \qquad (31)$$

and substituting (31)into (30) taking into account (30a) gives

$$ck/\omega \iint dx'dz'\, \rho(x',z')\, H_0^{(1)}(kr) = H_1^{(1)}(kr^{\circ})[\,\underline{v}_x(x_0,z_0)\,(x-x_0)/r^{\circ} + \underline{v}_z(x_0,z_0)\,(z-z_0)/r^{\circ}] \qquad (32)$$

with

$$r^{\circ} = [(x-x_0)^2 + (z-z_0)^2]^{1/2} \qquad (32a)$$

This result suggests to define $\rho(x',z')$ as

$$\rho(x',z') = \underline{\rho}(x'_0,z'_0)\,\delta(x'-x_0)\,\delta(z'-z_0) \qquad (33)$$

substituting (33) into (32) gives finally the relation between charge and current densities

$$ck/\omega\ \underline{\rho}(x_0,z_0)\, H_0^{(1)}(kr^{\circ}) = H_1^{(1)}(kr^{\circ})[\,\underline{v}_x(x_0,z_0)\,(x-x_0)/r^{\circ} + \underline{v}_z(x_0,z_0)\,(z-z_0)/r^{\circ}] \qquad (34)$$

while according to (27) and (34), the scalar potential ϕ becomes

$$\phi(x,z) = -i/4\varepsilon\ \underline{\rho}(x_0,z_0)\, H_0^{(1)}(kr^{\circ}) \qquad (35)$$

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and substituting (35) into (25) achieves to determine the electric components of theTM field.

III. FIELDS AND POTENTIALS IN CYLINDRICAL COORDINATES (r,θ z)

In cylindrical coordinates, the equations (1) become for harmonic fields

$$\mu H_r = 1/r\,\partial_\theta A_z - \partial_z A_\theta \quad , \quad E_r = -\partial_r\phi + i\omega/c\, A_r$$
$$\mu H_\theta = \partial_z A_r - \partial_r A_z \quad , \quad E_\theta = -\partial_\theta\phi + i\omega/c\, A_\theta$$
$$\mu H_z = 1/r\,\partial_r(rA_\theta) - 1/r\,\partial_\theta A_r \quad (36a) \quad , \quad E_r = -\partial_z\phi + i\omega/c\, A_z \quad (36b)$$

A. TE FIELD

We consider the TE field with components

$$E_\theta(r,z) \quad , \quad H_r(r,z) \quad , \quad H_z(r,z) \qquad (37a)$$

while potentials , charge and currents are

$$A_r = A_z = 0\ ,\ A_\theta(r,z)\ ,\ \phi = 0 \quad ; \quad J_r = J_z = 0,\ J_\theta(r,z) \quad ; \quad \rho = 0 \qquad (37b)$$

We get from (36a,b)

$$\mu H_r = -\partial_z A_\theta \quad , \quad \mu H_z = 1/r\,\partial_r(rA_\theta) \quad , \quad E_\theta = i\omega/c\, A_\theta \qquad (38)$$

and according to (2a), the inhomogeneous Helmholtz equation satisfied by A_θ is

$$1/r\,\partial_r(r\partial_r A_\theta) - 1/r^2 A_\theta + \partial_z^{\,2} A_\theta + k^2 A_\theta = -\mu J_\theta \qquad k^2 = \omega^2\varepsilon\mu/c^2 \tag{39}$$

Assuming that the current makes separable Eq.(39), A_θ and J_θ have the representation

$$A_\theta(r,z) = \alpha_\theta(r)\,\psi(z) \quad , \quad J_\theta(r,z) = \gamma_\theta(r)\,\psi(z) \tag{40}$$

with

$$\psi(z) = \int_k^\infty dk_z \cos[k_z(z-z')] \tag{40a}$$

Then, substituting (40) into (39) and taking into account (40a) give

$$(\partial_r^{\,2} + 1/r\partial_r - 1/r^2 - k_r^{\,2})\,\alpha_\theta(r) = -\mu\gamma_\theta(r) \quad , \quad k_r^{\,2} = k_z^{\,2} - k^2 > 0 \tag{41}$$

To solve (41), we use the Green function g(r,r') solution of the equation

$$(\partial_r^{\,2} + 1/r\partial_r - 1/r^2 - k_r^{\,2})\,g(r,r') = -4\pi/r\,\delta(r-r') \tag{42}$$

which is a particular case of an equation discussed by Schwinger [2] : its $-m^2/r^2$ term becomes here $-1/r^2$. Then, making m = 1 in its results gives

$$g(r,r') = 4\pi\, I_1(k_r r_<)\, K_1(k_r r_>) \tag{43}$$

in which I_1, K_1 are the modified Bessel and Hankel functions of first order while $r_<$, $r_>$ means respectively r-r' > 0, r-r' < 0. And, with (43) the solution of Eq.(41) is

$$\alpha_\theta(r) = -4\pi\mu\,[\,\int_0^r dr'\,\gamma_\theta(r')\,I_1(k_r r') + \int_r^\infty dr'\,\gamma_\theta(r')\,I_1(k_r r')\,K_1(k_r r')] \tag{44}$$

So, finally taking into account (40), 40a) and (44) we get

$$A_\theta(r,z) = -4\mu\pi\psi(z)\,[\,\int_0^r dr'\,\gamma_\theta(r')\,I_1(k_r r') + \int_r^\infty dr'\,\gamma_\theta(r')\,I_1(k_r r')\,K_1(k_r r')] \tag{45}$$

Suppose now the current $J_\theta(r,z)$ due to a moving point charge

$$J_\theta(r,z) = \psi(z)\,\underline{v}_\theta(r)\,\delta(r-r') \quad , \quad \underline{v}_\theta(r) = e\,v_\theta(r) \tag{46}$$

Then, according to (45)

$$A_\theta(r,z) = -4\mu\pi\,\psi(z)\,[\Omega_1(r) + \Omega_2(r)] \tag{47}$$

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$$\Omega_1(r) = \int_0^r dr'\,\underline{v}_\theta(r')\,I_1(k_r r')\,\delta r\text{-}r') = ½\, I_1(k_r r^-)\,\underline{v}_\theta(r^-) \tag{47a}$$

$$\Omega_2(r) = \int_r^\infty dr'\,\underline{v}_\theta(r')\,I_1(k_r r')\,K_1(k_r r')\;\;\delta r\text{-}r') = ½\, I_1(k_r r^+)\,K_1(k_r r^+)\;\,\underline{v}_\theta(r^+) \tag{47b}$$

in which $r^\pm = r \pm 0$. And, according to (38 and (47)

$$E_\theta(r,z) = -4i\mu\pi/c\;\psi(z)\,[\Omega_1(r) + \Omega_2(r)]$$
$$H_r(r,z) = 4\pi\psi'(z)\,[\Omega_1(r) + \Omega_2(r)]$$
$$H_z(r,z) = -4\pi\psi(z)\,[1/r\delta_r\{r\Omega_1(r)\} + 1/r\delta_r\{r\Omega_2(r)\}] \tag{48}$$

with in this last expression

$$1/r\delta_r\{rI_1(kr)\} = kI_0(kr) \quad , \quad 1/r\delta_r\{rK_1(kr)\} = kK_0(kr) \tag{49}$$

Remark 2. If the Green function is null on a cylinder of radius a then [2]

$$G(r,r') = i\pi/2\; I_1(k_r r_<)\,[K_1(k_r r_>) - I_1(k_r r_>)\,K_1(k_r a) / I_1(k_r a)] \tag{50}$$

Remark 3. $\psi(z)$ is a truncated delta function

$$\psi(z) = \delta(z\,) - 1/k\pi\,\sin(kz) \tag{51}$$

B. TM FIELDS

We consider the TM field with components

$$H_\theta(r,z) \quad , \quad E_r(r,z) \quad , \quad E_z(r,z) \tag{52a}$$

with the potentials, charge and currents

$$A_\theta = 0 \;,\; A_r(r,z) \;,\; A_z(r,z)\,, \quad \phi\,(r,z) \quad ;\; J_\theta = 0 \;,\; J_r(r,z) \;,\; J_z(r,z) \quad , \rho(r,z) \tag{52b}$$

We get from (36 a,b)

$$E_r = -\partial_r\phi + i\omega A_r/c \quad , \quad E_z = -\partial_z\phi + i\omega A_z/c \quad , \quad \mu H_\theta = \partial_z A_r - \partial_r A_z \tag{53}$$

According to (2a) the potentials A_r, A_z satisfy the Helmholtz equations

$$1/r\,\partial_r(r\partial_r A_r) - 1/r^2 A_r + \partial_z^{\,2} A_r + k^2 A_r = -\mu J_r \tag{54a}$$

$$1/r\,\partial_r(r\partial_r A_z) + \partial_z^{\,2} A_z + k^2 A_z = -\mu J_z \tag{54b}$$

in which $k^2 = \omega^2\varepsilon\mu/c^2$.

We see that Eq.(54a) has the same form as Eq.(39) for A_θ so that still assuming that the cur-rents make separable these inhomogeneous Helmholtz equations, we write

$$A_r(r,z) = \alpha_r(r)\,\psi(z) \quad , \quad J_r(r,z) = \gamma_r(r)\,\psi(z) \tag{55}$$

with $\psi(z)$ supplied by (40a) : we have just to change the subscript θ into r in all the expres-sions of Sec.3.1 to get the corresponding results for the potential A_r, in particular according to (45)

$$A_r(r,z) = -4\mu\pi\,\psi(z)\,[\,\int_0^r dr'\,\gamma_r(r')\,I_1(k_r r') + \int_r^\infty dr'\,\gamma_r(r')\,I_1(k_r r')\,K_1(k_r r')] \tag{56}$$

The additional term due to the scalar field in the expression (53) of $E_r(r,z)$ is discussed here below.

Now the Helmholtz equation (54b) for $A_z(r,z)$ is bit different and, with

$$A_z(r,z) = \alpha_z(r)\,\psi(z) \quad , \quad J_z(r,z) = \gamma_z(r)\,\psi(z) \qquad (57)$$

we get the differential equation

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$$(\partial_r^2 + 1/r\partial_r - k_r^2)\,\alpha_z(r) = -\mu\gamma_z(r) \quad , \qquad k_r^2 = k_z^2 - k^2 > 0 \qquad (58)$$

To solve (58), we look for the Green function solution of the equation

$$(\partial_r^2 + 1/r\partial_r - k_r^2)\,g(r,r') = -4\pi/r\;\delta(r-r') \qquad (59)$$

which is still a particular case : m = 0, of the Schwinger equation so that we have just to trans-form H_1, K_1, into H_0 , K_0 in (43) to get

$$g(r,r') = 4\pi\, I_0(k_r r_<)\, K_0(k_r r_>) \qquad (60)$$

Then, changing the subscript r into z on α,γ in (56) and I_1, K_1 into I_0, K_0 , we get

$$A_z(r,z) = -4\mu\pi\,\psi(z)\,[\,\int_0^r dr'\,\gamma_z(r')\,I_0(k_r r') + \int_r^\infty dr'\,\gamma_z(r')\,I_0(k_r r')\,K_0(k_r r')] \qquad (61)$$

which supplies at once $E_z(r,z)$ in (53) leaving aside for later $-\partial_z\phi$.

Now, to get $H_\theta(r,z)$, we note $A_{r,1}$ and $A_{z,0}$ the terms in the square bracket of Eqs.(56) and (61) so that :

$$A_r(r,z) = -4\mu\pi\,\psi(z)\,A_{r,1} \quad , \quad A_z(r,z) = -4\mu\pi\,\psi(z)\,A_{z,0} \qquad (62)$$

and, according to (53)

$$H_\theta(r,z) = -4\pi\,\psi'(z)\,A_{r,1} - 4\pi\,\psi(z)\,\partial_r A_{z,0} \qquad (63)$$

with

$$\partial_r A_{z,0} = \gamma_z(r)\,I_0(k_r r^-) - \gamma_z(r)\,I_0(k_r r^+)\,K_0(k_r r^+) \qquad (63a)$$

It is easy to transpose these results to situations where the components J_r, J_z of the current are generated by a moving point charge.

We now have to discuss the scalar potential $\phi(r,z)$ solution according to (2b) of the Hel-mholtz equation

$$1/r\,\partial_r(r\partial_r\phi) + \partial_z^2\phi + k^2\phi = -\rho/\varepsilon \qquad (64)$$

similar to (54b) with A_z and μJ_z changed into ϕ and ρ/ε. Then, with

$$\phi(r,z) = \chi(r)\,\psi(z) \quad , \quad \rho(r,z) = \sigma(r)\,\psi(z) \qquad (65)$$

the equation (64) becomes

$$[1/r\,\partial_r(r\partial_r) - k_r^2]\,\chi(r) = -\sigma(r/\varepsilon \quad , k_r^2 = k_z^2 - k^2 > 0 \qquad (66)$$

and, still using the Green function (60), the solution of Eq.(64) is similar to (61)

$$\phi(r,z) = -4\pi/\varepsilon\;\psi(z)\,[\,\int_0^r dr'\,\sigma(r')\,I_0(k_r r') + \int_r^\infty dr'\,\sigma(r')\,I_0(k_r r')\,K_0(k_r r')] \qquad (67)$$

Substituting (67) into (53) achieves to determine $E_r(r,z)$ and $E_z(r,z)$.

Now the constraint (2c) implies

$$i\omega\varepsilon\mu/c = -1/r\partial_r(rA_r) - \partial_z A_z \qquad (68)$$

To avoid very intricate calculations, we do not check the consequences of (68).

IV. DISCUSSION

We have analyzed in a isotropic homogeneous medium one of the three possible sets of TE and TM electromagnetic waves, obtained by a circular permutation of the coordinates x,y,z or r,θ,z in cartesian and cylindrical ccordinates respectively. In any case, one has to do with a 2D problem and, working with scalar or vector potentials leads to 2D inhomogeneous Hel- 8 mholtz equations (fields and potentials are assumed harmonic). For cartesian coordinates. all these three possible Helmholtz equations have the same form so, it does not matter to assign x or y or z to a particular component of fields and potentials. The situation is different in cy-lindrical coordinates : first Helmholtz equation differs for each component of the vector po-tential **A** and it is not indifferent to assign r or θ or z to a particular component of **A**, Second the current **J**(r,z) = $\mathbf{\Gamma}$(r) $\psi(z)$ makes separable the inhomogeneous Helmholtz equation and $\psi(z)$ has a particular behaviour : according to (40a) $\lim_{k\Rightarrow 0}\psi(z) = \delta(z)$ while according to (51) for $k > 0$ $\psi(z)$ appears as a sinc function of k. Note that TM fields require in any situa-tion.more elaborate calculations.

It is not easier to solve the 2D inhomogeneous Helmholtz equation than the 3D one, as shown by the solution due to Volterra [1] of the 2D wave equation. So, it is not evident à priori that to work with Green functions would perform better than solving the problem di-rectly. It has been known for a long time [1-3] that the 2D Green function of the Helmholtz equation in cartesian coordinates is the Hankel function (against exp(ikR)/R in 3D). But the 2D Green function used here for the radial component of the cylindrical coordinates does not seem to be well known [see nevertheless (2a)] although this function is a particular case of a Green function discussed by Schwinger [2] in a different context.

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As a result, this analysis of potentials for TE, TM fields in cartesian and cylindrical coordi-nates (in this last case with the constraints imposed on the currents) provides rather manage-able expressions of the electric and magnetic fields made explicit for currents generated by moving point charges. Incidently, note that the 2D charge density has not the same dimension as the 3D one.To analyze the propagation of electromagnetic fields in different media ; anisotropic, inho-mogeneous, periodic dielecteric, metamaterials, photonic crystals, gratings… it is usual to consider situations where TE and TM fields can be discussed separately with the implicit idea that this analysis is easier in 2D than in 3D.This is true for numerical approaches of these problems since less work is required from computers but is not true to obtain analytical re-sults. So the question : why to persist in the research of analytical solutions ? First because, on one hand they make easier a physical interpretation of the results and also because one may generally obtain the order of their validity when they require approximations. Static and moving charges (currents), solutions of inhomogeneous wave equations, are res-pectively the sources of the electric and magnetic components of the electromagnetic field. But, they also may be considered as the sources of the scalar and vector components of the electromagnetic potential which appears in the Lorentz gauge as more fundamental as fields, a fact which is general according to [5] But what does a current such as $\psi(z)\,\Gamma(r)$ represent ? If $\psi(z) = \delta(z)$, one has to do with a charge moving in the $z = 0$ plane and its amplitude de-pends only on the distance to the origin of this plane. Now, if $\psi(z) \neq \delta(z)$, then according to (51), the charge is oscillating harmonically around the $z = 0$ plane.,

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