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MHD Oblique Stagnation-point Flow and Heat Transfer of a Micro polar Fluid towards to a Moving Plate with Radiation

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Abstract— This paper investigates the oblique stagnation-point flow and heat transfer of a micro polar fluid towards to a moving plate with radiation in the presence of magnetic field aligned with velocity slip on the boundary. A transformation reduces the differential equations to a set of ordinary ones and solved the problem by means of the Homotopy Analysis Method (HAM). The paper also discussed the effects of different parameters, especially the material parameter, magnetic parameter, radiation parameter and velocity slip factor on velocity, angular velocity, and temperature profiles.

***Key Words*—Micro polar Fluid, Oblique Stagnation Point, HAM, Heat Transfer.**

I. INTRODUCTION

Stagnation point flow problem has attracted great interest of scientists in the past few years, due to its application in chemical and manufacturing processes, for instance polymer extrusion, continuous casting of metals, wire drawing, glass blowing. Reza and Gupta [1] analyzed the steady two-dimensional oblique stagnation point flow on a surface which is stretched with a velocity proportional to the distance from a fixed point. Mahapatra et al. [2] discussed the oblique stagnation-point flow of an incompressible viscous-elastic fluid above a stretching surface and took the thermal boundary layer into account. Lok et al. [3] considered the problem of Non-orthogonal stagnation-point flow under the classical modified Hiemenz mathematic model for a micro polar fluid. Magneto hydrodynamic oblique stagnation-point flow was investigated by Grosan et al. [4]. Then, Li et al. [5] studied heat transfer of the oblique stagnation-point viscoelastic fluid. And the oblique stagnation slip flow of a micro polar fluid was studied by Lok et al. [6] and the problem was solved numerically using the Keller box method. Later, Husain et al. [7] solved the problem of the two-dimensional oblique stagnation point flow on a stretching surface in a viscoelastic fluid under the assumption that the fluid impinges on the wall obliquely. Mahapatra et al. [8] investigated the oblique stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. Mahmoud et al. [9] considered the problem of MHD stagnation point flow of a micro polar fluid towards a moving surface with radiation. This paper are now investigate the problem of the MHD oblique stagnation-point flows and heat transfer of a micro polar fluid with velocity slip and thermal radiation, and solved the problem by means of Homotopy Analysis Method (HAM).

II. MATHEMATICAL FORMULATION

Consider the steady two-dimensional flow of a micro polar fluid near an oblique stagnation point on a plat coinciding with the plane $Y=0$, the X-axis is along the surface and the Y-axis is perpendicular to it and the flow being confined in the region $Y>0$. Cartesian coordinates (X,Y) are taken. The governing equations for the problem are written as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial X}+\left(\frac{\mu+\kappa}{\rho}\right)\nabla^2U+\frac{\kappa}{\rho}\frac{\partial N}{\partial Y}-\frac{\sigma B_0^{\ 2}}{\rho}U \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\left(\frac{\mu+\kappa}{\rho}\right)\nabla^2V-\frac{\kappa}{\rho}\frac{\partial N}{\partial X} \tag{3}$$

$$\rho j\left(U\frac{\partial N}{\partial X}+V\frac{\partial N}{\partial Y}\right)=\gamma\nabla^2N-\kappa\left(2N+\frac{\partial U}{\partial Y}-\frac{\partial V}{\partial X}\right) \tag{4}$$

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$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{\kappa}{\rho c_p}\nabla^2 T-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial Y} \quad (5)$$

With boundary conditions

$$Y=0: U=\gamma\frac{\partial U}{\partial Y}, V=0, N=-n\left(\frac{\partial U}{\partial Y}-\frac{\partial V}{\partial X}\right), T=T_w \quad (6)$$

$$Y\to\infty: U=U_e(X,Y)=aX+bY, v=V_e(Y)=-aY, N=-\frac{b}{2}, T\to T_\infty \quad (7)$$

For large Y, (2) and (3) become

$$-\frac{1}{\rho}\frac{\partial P}{\partial X}=U_e\frac{\partial U_e}{\partial X}+V_e\frac{\partial U_e}{\partial Y}+\frac{\sigma B_0^2}{\rho}U_e \quad (8)$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial Y}=V_e\frac{\partial V}{\partial Y} \quad (9)$$

Using (8) and (9), (2) and (3) can be written as

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=U_e\frac{\partial U_e}{\partial X}+V_e\frac{\partial U_e}{\partial Y}+\left(\frac{\mu+\kappa}{\rho}\right)\nabla^2 U+\frac{\kappa}{\rho}\frac{\partial N}{\partial Y}+\frac{\sigma B_0{}^2}{\rho}(U_e-U) \quad (10)$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial V}{\partial Y}=V_e\frac{\partial V}{\partial Y}+\left(\frac{\mu+\kappa}{\rho}\right)\nabla^2 V-\frac{\kappa}{\rho}\frac{\partial N}{\partial X} \quad (11)$$

Where U and V are the velocity components along the X - and Y - axes, T is the temperature, N is the component of the micro rotation vector (or angular velocity) normal to the $X-Y$ plane respectively.
Using the Roseland approximation [9], we have

$$q_r=-\frac{4\sigma_0}{3\kappa_0}\frac{\partial T^4}{\partial Y} \quad (12)$$

Where, σ_0 is the Stefan-Boltzmann constant and κ_0 is the mean absorption coefficient.

Using the following non-dimensional variables ($l=\left(\nu/a\right)^{1/2}$, $U_0=(a\nu)^{1/2}$)

$$x=X/l, y=Y/l, u=U/U_0, v=V/U_0, u_e=U_e/U_0, v_e=V_e/U_0, N=\left(\nu/U_0^2\right)N \quad (13)$$

The following non-dimensional equations can be obtained

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (14)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+v_e\frac{\partial u_e}{\partial y}+(1+K)\nabla^2 u+K\frac{\partial N}{\partial y}+M(u_e-u) \quad (15)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=v_e\frac{\partial v_e}{\partial y}+(1+K)\nabla^2 v-K\frac{\partial N}{\partial x} \quad (16)$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=(1+K/2)\nabla^2 N-K(2N+\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}) \quad (17)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\Pr}\left(1+\frac{4}{3R}\right)\nabla^2 T \quad (18)$$

The boundary conditions become

$$y=0: u=\gamma\frac{\partial u}{\partial y}, v=0, N=-n\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right), T=T_w \quad (19)$$

$$y\to\infty: u=u_e(x,y)=x+2\alpha y, v=v_e(y)=-y, N=N_e=-\alpha, T\to T_\infty \quad (20)$$

Where $K=\frac{\mu}{\kappa}$ is the micro polar material parameter, μ is the dynamic viscosity, κ is vortex viscosity, $M=\frac{\sigma B_0^2}{\rho b}$ is the magnetic parameter, and ∇^2 is the two-dimensional Laplace. $\Pr=\frac{\nu\rho c_p}{\kappa}$ is the Prantle number, γ is the

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dimensionless velocity slip factor, $R=\dfrac{\kappa_0\kappa}{4\sigma_0 T_\infty^3}$ is the radiation parameter, α is the shear flow parameter n is a constant such that $0\le n\le 1$, the strong concentration case $n=0$ represents the concentrated particle flows in which the microelements close to the wall surface and unable to rotate. The weak concentration case $n=0.5$ indicates the vanishing of the anti-symmetrical part of the stress tensor. The case $n=1$ is used for the modeling of turbulent boundary layer flows. In this paper, we consider the cases of $n=0$ (strong concentration) and $n=0.5$ (weak concentration) [6].

Proceeding with the analysis, we define a stream function $\psi(x,y)$ such that:

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x} \tag{21}$$

$$\psi(x,y)=xf(y)+g(y),N(x,y)=xs(y)+t(y),\theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{22}$$

The continuity equation (14) is automatically satisfied, (15)-(18) can be reduced to the following ordinary differential equations.

$$(1+K)f'''+ff''+Ks'-(f')^2-Mf'+(1+M)=0 \tag{23}$$

$$(1+K)g'''+fg''+2M\alpha(K+1)f''+Kt'-f'g'-Mg'+2M\alpha ff'+2KM\alpha s=0 \tag{24}$$

$$(0.5K+1)s''-Kf''+fs'-sf'-2Ks=0 \tag{25}$$

$$(0.5K+1)t''-Kg''+ft'-sg'-2Kt=0 \tag{26}$$

$$\left(1+\frac{4}{3R}\right)\theta''+\Pr f\theta'=0 \tag{27}$$

Subject to the following boundary conditions:

$$f(0)=0,s(0)=-nf''(0),\ f'(0)=\gamma f''(0),f'(\infty)=1,s(\infty)=0,\theta(0)=1,\theta(\infty)=0 \tag{28}$$

$$g(0)=0,t(0)=-ng''(0),g'(0)=\gamma g''(0),g''(\infty)=2\alpha,t(\infty)=-\alpha \tag{29}$$

If we take $g'(y)=2\alpha\Phi(y)$ and $t(y)=2\alpha\Gamma(y)$, then (24) and (26) become

$$(1+K)\Phi''+f\Phi'+(1+K)Mf''+K\Gamma'-f'\Phi-M\Phi+Mff'+KMh=0 \tag{30}$$

$$(0.5K+1)\Gamma''-K\Phi'+f\Gamma'-h\Phi-2K\Gamma=0 \tag{31}$$

Subject to the boundary conditions (29) which become

$$\Phi(0)=\Phi'(0),\Gamma(0)=-n\Phi'(0),\ \Phi'(\infty)=1,\ \Gamma(\infty)=-0.5 \tag{32}$$

III. Application of HAM

A. Zeroth-order deformation equations

According to the HAM, which is mentioned by Liao [10]-[15], we can approximate nonlinear problem more efficiently by choosing a proper set of base functions to ensure its convergence. In this paper, we choose the following set of base functions to express $f(\eta)$, $\Phi(\eta)$, $\Gamma(\eta)$, and $\theta(\eta)$ as:

$$\{\eta,e^{-\eta}\} \tag{33}$$

According to the rule of solution expression, the initial approximations and auxiliary linear operators can be chose in the following form [16]-[18]:

$$f_0(\eta)=-\frac{1}{1+\gamma}+\eta+\frac{e^{-\eta}}{1+\gamma},s_0(\eta)=-\frac{n}{1+\gamma}e^{-\eta},\ \Phi_0(\eta)=\eta+\gamma,\Gamma_0(\eta)=-\frac{1}{2}+(\frac{1}{2}-n)e^{-\eta},\ \theta_0(\eta)=e^{-\eta} \tag{34}$$

$$L_f(\phi)=\frac{\partial^3\phi}{\partial\eta^3}+\frac{\partial^2\phi}{\partial\eta^2},L_s(\Psi)=\frac{\partial^2\Psi}{\partial\eta^2}+\frac{\partial\Psi}{\partial\eta},L_\Phi(\Omega)=\frac{\partial^2\Omega}{\partial\eta^2},L_\Gamma(\Xi)=\frac{\partial^2\Xi}{\partial\eta^2}+\frac{\partial\Xi}{\partial\eta},L_\theta(\Theta)=\frac{\partial^2\Theta}{\partial\eta^2}+\frac{\partial\Theta}{\partial\eta} \tag{35}$$

$$L_\theta(C_1+C_2e^{-\eta})=0,\ L_f\left(C_1+C_2\eta+C_3e^{-\eta}\right)=0,\ L_s(C_1+C_2e^{-\eta})=0,L_\Phi(C_1+C_2\eta)=0,\ L_\Gamma(C_1+C_2e^{-\eta})=0 \tag{36}$$

C_1, C_2 and C_3 is any constants. Furthermore, the nonlinear operators are defined by the following forms:

$$N_f[\phi(\eta;q),\Psi(\eta;q)]=(1+K)\frac{\partial^3\phi}{\partial\eta^3}+\phi\frac{\partial^2\phi}{\partial\eta^2}+K\frac{\partial\Psi}{\partial\eta}-(\frac{\partial\phi}{\partial\eta})^2-M\frac{\partial\phi}{\partial\eta}+(1+M) \tag{37}$$

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$$N_{\Phi}[\Omega(\eta;q),\phi(\eta;q),\Xi(\eta;q),\Psi(\eta;q)]=(1+K)\frac{\partial^2\Omega}{\partial\eta^2}+\phi\frac{\partial\Omega}{\partial\eta}$$
$$+(1+K)M\frac{\partial^2\phi}{\partial\eta^2}+K\frac{\partial\Xi}{\partial\eta}-\frac{\partial\phi}{\partial\eta}\Omega-M\Omega+M\phi\frac{\partial\phi}{\partial\eta}+KM\Psi \tag{38}$$

$$N_s[\Psi(\eta;q),\phi(\eta;q)]=(\frac{1}{2}K+1)\frac{\partial^2\Psi}{\partial\eta^2}-K\frac{\partial^2\phi}{\partial\eta^2}+\phi\frac{\partial\Psi}{\partial\eta}-\Psi\frac{\partial\phi}{\partial\eta}-2K\Psi \tag{39}$$

$$N_{\Gamma}[\Xi(\eta;q),\Omega(\eta;q),\phi(\eta;q),\Psi(\eta;q)]=(\frac{1}{2}K+1)\frac{\partial^2\Xi}{\partial\eta^2}-K\frac{\partial\Omega}{\partial\eta}+\phi\frac{\partial\Xi}{\partial\eta}-\Psi\Omega-2K\Xi \tag{40}$$

$$N_{\theta}[\Theta(\eta;q),\phi(\eta;q)]=\left(1+\frac{4}{3R}\right)\frac{\partial^2\Theta}{\partial\eta^2}+\Pr\phi\frac{\partial\Theta}{\partial\eta} \tag{41}$$

And then construct the following equations as the zeroth-order deformation equations:

$$(1-q)L_f[\phi(\eta;q)-f_0(\eta)]=qh_fH_f(\eta)N_f[\phi(\eta;q),\Omega(\eta;q),\Psi(\eta;q),\Xi(\eta;q),\Theta(\eta;q)] \tag{42}$$

$$(1-q)L_{\Phi}[\Omega(\eta;q)-\Phi_0(\eta)]=qh_{\Phi}H_{\Phi}(\eta)N_{\Phi}[\phi(\eta;q),\Omega(\eta;q),\Psi(\eta;q),\Xi(\eta;q),\Theta(\eta;q)] \tag{43}$$

$$(1-q)L_{\Gamma}[\Xi(\eta;q)-\Gamma_0(\eta)]=qh_{\Gamma}H_{\Gamma}(\eta)N_{\Gamma}[\phi(\eta;q),\Omega(\eta;q),\Psi(\eta;q),\Xi(\eta;q),\Theta(\eta;q)] \tag{44}$$

$$(1-q)L_{\theta}[\Theta(\eta;q)-\theta_0(\eta)]=qh_{\theta}H_{\theta}(\eta)N_{\theta}[\phi(\eta;q),\Omega(\eta;q),\Psi(\eta;q),\Xi(\eta;q),\Theta(\eta;q)] \tag{45}$$

With the following boundary conditions

$$\phi(0,\eta)=0,\Psi(0,q)=-n\frac{\partial^2\phi}{\partial\eta^2}\bigg|_{(0,q)},\frac{\partial\phi}{\partial\eta}\bigg|_{(0,q)}=\gamma\frac{\partial^2\phi}{\partial\eta^2}\bigg|_{(0,q)},\frac{\partial\phi}{\partial\eta}\bigg|_{(\infty,q)}=1,\Psi(\infty,q)=0,$$
$$\Theta(0,q)=1,\Omega(0,q)=\frac{\partial\Omega}{\partial\eta}\bigg|_{(0,q)},\Xi(0,q)=-n\frac{\partial\Omega}{\partial\eta}\bigg|_{(0,q)},\frac{\partial\Omega}{\partial\eta}\bigg|_{(\infty,q)}=1,\Xi(\infty,q)=-\frac{1}{2} \tag{46}$$

When $q=0$ and $q=1$

$$\phi(\eta,0)=f_0(\eta),\phi(\eta,1)=f(\eta),\Psi(\eta,0)=s_0(\eta),\Psi(\eta,1)=s(\eta),\Theta(\eta,0)=\theta_0(\eta),$$
$$\Theta(\eta,1)=\theta(\eta),\Omega(\eta,0)=\Phi_0(\eta),\ \ \Omega(\eta,1)=\Phi(\eta),\Xi(\eta,0)=\Gamma_0(\eta),\Xi(\eta,1)=\Gamma(\eta) \tag{47}$$

When q increases 0 to 1, and then $f(\eta)$, $s(\eta)$, $\theta(\eta)$, $\Phi(\eta)$, $\Gamma(\eta)$ vary form $f_0(\eta)$, $s_0(\eta)$, $\theta_0(\eta)$, $\Phi_0(\eta)$, $\Gamma_0(\eta)$ to $f(\eta)$, $s(\eta)$, $\theta(\eta)$, $\Phi(\eta)$ $\Gamma(\eta)$.According to the Taylor's theorem, $\phi(\eta,q)$, $\Psi(\eta,q)$, $\Theta(\eta,q)$, $\Omega(\eta,q)$, $\Xi(\eta,q)$ can be expanded in a series of q as follows:

$$\phi(\eta,q)=f_0+\sum_{m=1}^{\infty}f_m(\eta)q^m,\ f_m(\eta)=\frac{1}{m!}\frac{\partial^m\phi(\eta,q)}{\partial q^m}\bigg|_{q=0} \tag{48}$$

$$\Psi(\eta,q)=s_0+\sum_{m=1}^{\infty}s_m(\eta)q^m,s_m(\eta)=\frac{1}{m!}\frac{\partial^m\Psi(\eta,q)}{\partial q^m}\bigg|_{q=0} \tag{49}$$

$$\Theta(\eta,q)=\theta_0+\sum_{m=1}^{\infty}\theta_m(\eta)q^m,\theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\Theta(\eta,q)}{\partial q^m}\bigg|_{q=0} \tag{50}$$

$$\Omega(\eta,q)=\Phi_0+\sum_{m=1}^{\infty}\Phi_m(\eta)q^m,\Phi_m(\eta)=\frac{1}{m!}\frac{\partial^m\Omega(\eta,q)}{\partial q^m}\bigg|_{q=0} \tag{51}$$

$$\Xi(\eta,q)=\Gamma_0+\sum_{m=1}^{\infty}\Gamma_m(\eta)q^m,\Gamma_m(\eta)=\frac{1}{m!}\frac{\partial^m\Xi(\eta,q)}{\partial q^m}\bigg|_{q=0} \tag{52}$$

If h_f, h_s, h_θ, h_Φ, h_Γ is properly chosen, the series (48)-(52) are convergent at $q=1$. Then, the solution series are:

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta),s(\eta)=s_0(\eta)+\sum_{m=1}^{\infty}s_m(\eta),(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta)$$
$$\Phi(\eta)=\Phi_0(\eta)+\sum_{m=1}^{\infty}\Phi_m(\eta),\Gamma(\eta)=\Gamma_0(\eta)+\sum_{m=1}^{\infty}\Gamma_m(\eta) \tag{53}$$

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B. High-order deformation equations

Define the sets:

$$\begin{aligned} & f_m(\eta)=\{f_1(\eta), f_2(\eta)\cdots f_m(\eta)\},\ s_m(\eta)=\{s_1(\eta), s_2(\eta)\cdots s_m(\eta)\}, \\ & \Phi_m(\eta)=\{\Phi_1(\eta),\Phi_2(\eta)\cdots\Phi_m(\eta)\},\ \Gamma_m(\eta)=\{\Gamma_1(\eta),\Gamma_2(\eta)\cdots\Gamma_m(\eta)\}, \\ & \theta_m(\eta)=\{\theta_1(\eta),\theta_2(\eta)\cdots\theta_m(\eta)\} \end{aligned} \tag{54}$$

And then the mth-order deformation equations:

$$L_f[f_m(\eta)-\chi_m f_{m-1}(\eta)]=h_f H_f(\eta)R_m^f(\eta) \tag{55}$$

$$L_s\left[s_m(\eta)-\chi_m s_{m-1}(\eta)\right]=h_s H_s(\eta)R_m^s(\eta) \tag{56}$$

$$L_\Phi\left[\Phi_m(\eta)-\chi_m\Phi_{m-1}(\eta)\right]=h_\Phi H_\Phi(\eta)R_m^\Phi(\eta) \tag{57}$$

$$L_\Gamma\left[\Gamma_m(\eta)-\chi_m\Gamma_{m-1}(\eta)\right]=h_\Gamma H_\Gamma(\eta)R_m^\Gamma(\eta) \tag{58}$$

$$L_\theta\left[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)\right]=h_\theta H_\theta(\eta)R_m^\theta(\eta) \tag{59}$$

Subject to the following boundary conditions:

$$\begin{aligned} & f_m(0)=0,\quad s_m(0)=-nf_m''(0),\ f_m'(0)=f_m''(0),\ f_m'(\infty)=0,\ s_m(\infty)=0,\theta(0)=0, \\ & \theta(\infty)=0,\Phi_m(0)=\Phi_m'(0),\ \Gamma_m(0)=-n\Phi_m'(0),\ \Phi_m'(\infty)=0,\ \Gamma_m(\infty)=0 \end{aligned} \tag{60}$$

Under the conditions

$$R_m^f(\eta)=(1+K)f_{m-1}'''(\eta)+\sum_{k=0}^{m-1}f_k(\eta)f_{m-1-k}''(\eta)+Ks'(\eta)-\sum_{k=0}^{m}f_k'(\eta)f_{m-1-k}'(\eta)-Mf_{m-1}'(\eta)+(1+M) \tag{61}$$

$$\begin{aligned} R_m^\Phi &=(1+K)\Phi_{m-1}''+\sum_{k=0}^{m-1}f_k(\eta)\Phi_{m-1-k}'(\eta)+(1+K)Mf_{m-1}''(\eta)+K\Gamma_{m-1}'(\eta) \\ &-\sum_{k=0}^{m-1}f_k'(\eta)\Phi_{m-1-k}(\eta)-M\Phi_{m-1}(\eta)+M\sum_{k=0}^{m-1}f_k(\eta)f_{m-1-k}'(\eta)+KMs(\eta) \end{aligned} \tag{62}$$

$$R_m^s=(\frac{1}{2}K+1)s_{m-1}''(\eta)-Kf_{m-1}''(\eta)+\sum_{k=0}^{m-1}f_k(\eta)s_{m-1-k}'(\eta)-\sum_{k=0}^{m-1}s_k(\eta)f_{m-1-k}'(\eta)-2Ks_{m-1}(\eta) \tag{63}$$

$$R_m^\Gamma=(\frac{1}{2}K+1)\Gamma_{m-1}''(\eta)-K\Phi_{m-1}'(\eta)+\sum_{k=0}^{m-1}f_k(\eta)\Gamma_{m-1-k}'(\eta)-s(\eta)\Phi_{m-1}(\eta)-2K\Gamma_{m-1}(\eta) \tag{64}$$

$$R_m^\theta=\left(1+\frac{4}{3R}\right)\theta_{m-1}''(\eta)+\Pr\sum_{k=0}^{m-1}f_k(\eta)\theta_{m-1-k}'(\eta) \tag{65}$$

And

$$\chi_m=\begin{cases}0, m\le 1\\ 1, m>1\end{cases} \tag{66}$$

According to the Rule of Solution Expression and Rule of Solution Existence described by Liao, we can choose

$$H_f(\eta)=H_s(\eta)=H_\theta(\eta)=H_\Phi(\eta)=H_\Gamma(\eta)=e^{-\eta} \tag{67}$$

According to the above equations and boundary layer, we can get the f_1 , h_1, θ_1, Φ_1, Γ_1 when $m=1$.

$$\begin{aligned} f_1 &=-e^{-2\eta}h_f[\frac{\eta+3+M+K(n-1)}{4(\gamma+1)}-\frac{1}{4(\gamma+1)^2}]+e^{-\eta}h_f[\frac{K(n-1)+M+2}{(\gamma+1)}-\frac{2K(n-1)+2M+7}{4(\gamma+1)^2}+\frac{1}{2(\gamma+1)^3}] \\ &-h_f[\frac{3K(n-1)+3M+5}{4(\gamma+1)}-\frac{K(n-1)+M+3}{2(\gamma+1)^2}+\frac{1}{2(\gamma+1)^3}] \end{aligned} \tag{68}$$

$$\begin{aligned} \Phi_1 &=h_\Phi\{\frac{(2\gamma+1)[4(\gamma-1)+2M(2\gamma+1)-K(2n-1)]}{8}+\frac{[4+3K(2\gamma+1)(n-1)]M}{12(\gamma+1)}+\frac{M}{36(\gamma+1)^2}\} \\ &-e^{-\eta}h_\Phi\frac{(\gamma^2+\gamma+1)(M+1)}{(\gamma+1)}-e^{-2\eta}h_\Phi[\frac{K(1-2n)}{8}+\frac{KM(n-1)-(\gamma+\eta+2)}{4(\gamma+1)}+\frac{M(\gamma\eta-\gamma+\eta-2)}{4(\gamma+1)^2}]-e^{-3\eta}h_\Phi\frac{M}{9(\gamma+1)^2} \end{aligned} \tag{69}$$

$$\theta_1=e^{-\eta}h_\theta\frac{R(9\gamma+5)\Pr-(6R+8)(\gamma+1)}{12(\gamma+1)R}+e^{-2\eta}h_\theta\frac{(8+6R)(\gamma+1)-3R\Pr[(3\gamma+1)+2(\gamma+1)\eta]}{12(\gamma+1)R}-e^{-3\eta}h_\theta\frac{\Pr}{6(\gamma+1)} \tag{70}$$

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$$s_1 = \mathrm{e}^{-2\eta} h_s \{ \frac{K(-2+3n)}{4(1+\gamma)} + \frac{n[1+3\gamma+2(1+\gamma)\eta]}{4(1+\gamma)^2} \} + \mathrm{e}^{-\eta} h_s \{ \frac{K(2-5n+2n^2+2\gamma-3n\gamma)}{4(1+\gamma)^2} + \frac{n[2M(1+\gamma)-\gamma(1+3\gamma)]}{4(1+\gamma)^3} \} \quad (71)$$

$$\Gamma_1 = -e^{-\eta} h_\Gamma [\frac{1-2n+K(8\eta-3)+4K(2n-n^2)+8n(M+1)\gamma}{8} + \frac{2+6KMn(n-1)+n(3M+11)}{12(\gamma+1)} - \frac{nM}{6(\gamma+1)^2}]$$
$$+ e^{-2\eta} h_\Gamma [\frac{(2\eta+1)(2n+1)+3K(2n-1)}{8} - \frac{1}{4(\gamma+1)}] + e^{-3\eta} h_\Gamma \frac{(2n-1)}{12(\gamma+1)} \quad (72)$$

When $m=1$ or $m=2$, in the same method, we can get f_2, s_2, θ_2, Φ_2, Γ_2 and f_3, s_3, θ_3, Φ_3, Γ_3 and so on. Taking into account the length of the formula, we omitted it in this paper.

IV. Convergence Of HAM Solutions

According to the HAM pointed out by Liao [10]-[15], the convergence and the rate of approximation for the HAM solution strongly depends on the value of auxiliary parameters. In this paper, we can choose $h_f = h_s = h_\theta = h_\Phi = h_\Gamma = h$ by means of the **Figure.1**. We can choose one of the proper values of h which is parallel to the horizontal axis to ensure that the solution is convergent. The value of h is chosen by $h=-0.4$. **Table.1** illustrates that the analytic solutions coincides well with the results of Lok et al. [6] when we choose $h=-0.4$.

Table 1 Initial values for $f'''(0)$ as a function of velocity slip factor γ and material parameter K when $n=0$ (strong concentration), $M=0$, $\alpha=1$, $\Pr=0$, $R=5$, $h=-0.4$

γ	K=0		K=1		K=3	
	Present Results	Lok et al. [6]	Present Results	Lok et al. [6]	Present Results	Lok et al. [6]
0.0	1.226680	1.23259	0.841207	0.84107	0.554429	0.55537
0.2	1.038180	1.04258	0.752172	0.75238	0.517929	0.51752
0.4	0.884420	0.88635	0.671666	0.67204	0.481254	0.48091
0.6	0.763849	0.76428	0.602450	0.60286	0.447019	0.44684
0.8	0.669241	0.66897	0.543924	0.54432	0.415862	0.41582
1.0	0.594019	0.59346	0.494523	0.49487	0.387828	0.38787
1.4	0.483383	0.48265	0.416837	0.41711	0.340247	0.34039
1.5	0.461552	0.46095	0.400866	0.40112	0.329906	0.33006

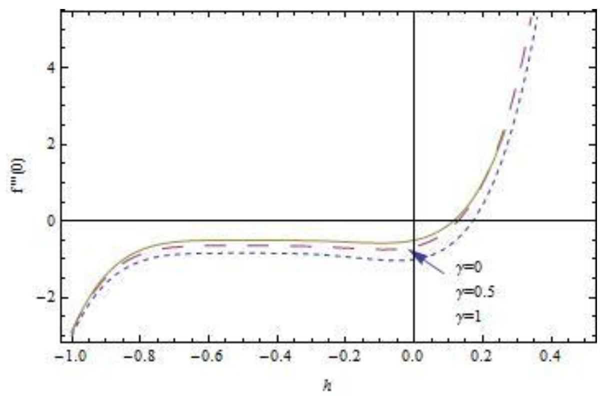

Fig. 1 The $f'''(0)-h$ curves by the fifth-order approximation for various γ when $M=1, K=1, n=0.5, \alpha=1, \Pr=1, \mathrm{R}=5$

V. Results And Discussion

The effects of γ, K, M, $\Pr$, R on the temperature profiles are shown in Figs. 2-6. **Fig.2** shows that temperature increases as K increases. But as γ, M, $\Pr$, R increase, it results in thickening boundary layer from **Figs. 3-6**. Figs. 7-14 show the effects of γ, M, K on the velocity (f' and g'), the angular velocity of microstructure ($-h$

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and t). It is observe that as γ increases, f'(**Fig. 7**) and g'(**Fig. 10**) increase while $-h$ (**Fig 12**)and t (**Fig 14**)decrease. Also, f' (**Fig 8**) increases, but g' (**Fig 11**) and $-h$ (**Fig 13**) decrease with increasing M . Fig. 9 illustrates that the increasing K decreases the velocity f'.

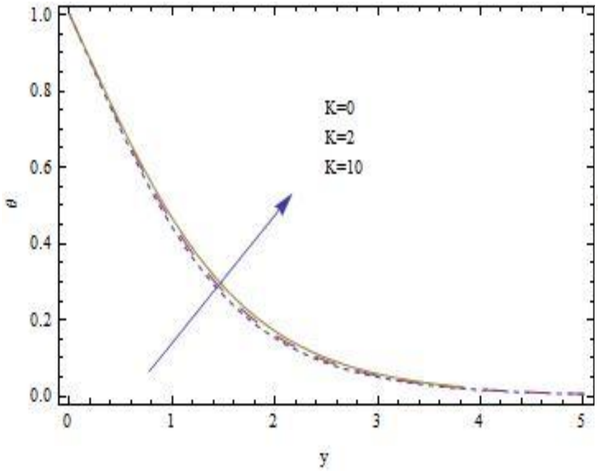

Fig. 2 Temperature profiles $\theta(y)$ for different values of K when $M=1, n=0, \alpha=1, \Pr=1, R=5, \gamma=0.5$

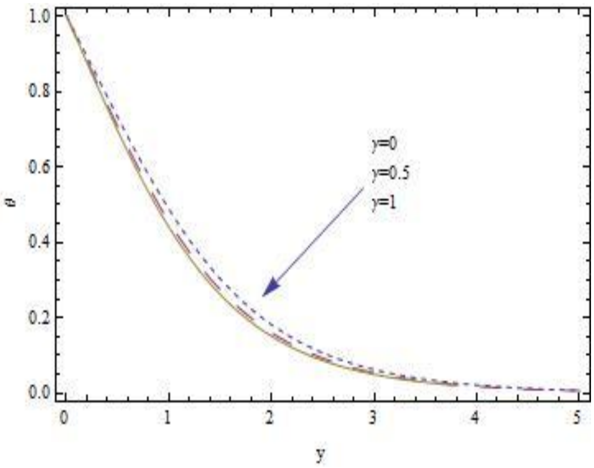

Fig. 3 Temperature profiles $\theta(y)$ for different values of γ when $M=1, K=1, n=0, \alpha=1, \Pr=1, \mathrm{R}=5$

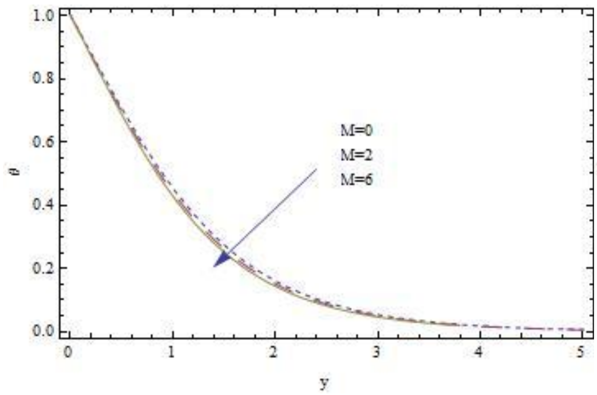

Fig. 4 Temperature profiles $\theta(y)$ for different values of M when $n=0, \alpha=1, \Pr=1, R=5, \gamma=0.5, K=0.5$

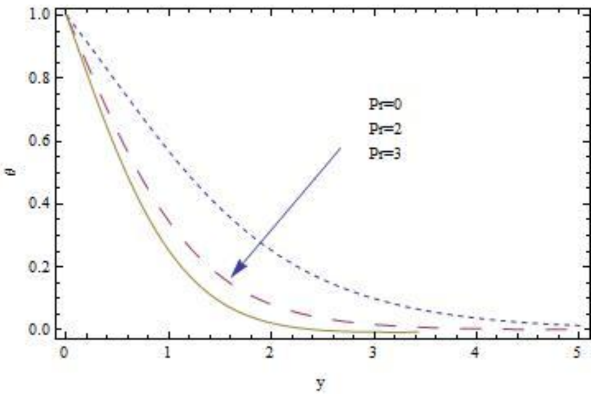

Fig. 5 Temperature profiles $\theta(y)$ for different values of $\Pr$ when $n=0, \alpha=1, R=5, \gamma=0.5, K=0.5, M=0.5$

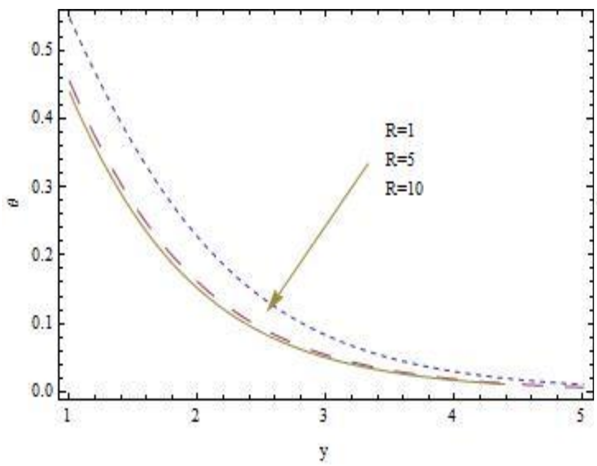

Fig. 6 Temperature profiles $\theta(y)$ for different values of R when $M=1, \mathrm{K}=1, n=0, \alpha=1, \Pr=1, \gamma=0.5$

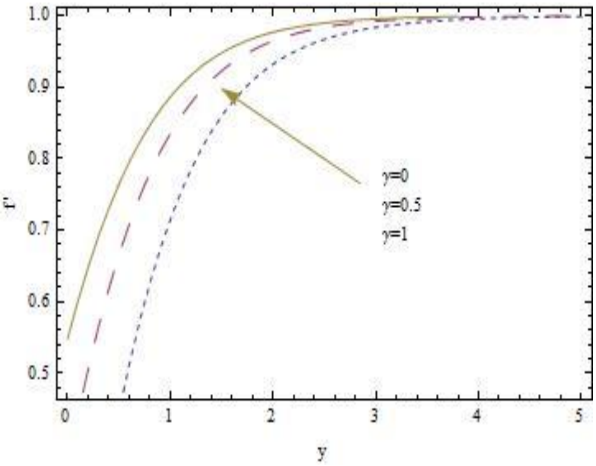

Fig. 7 Dimensionless velocity f' for different values of γ when $M=1, K=1, n=0, \alpha=1, \Pr=1, R=5$

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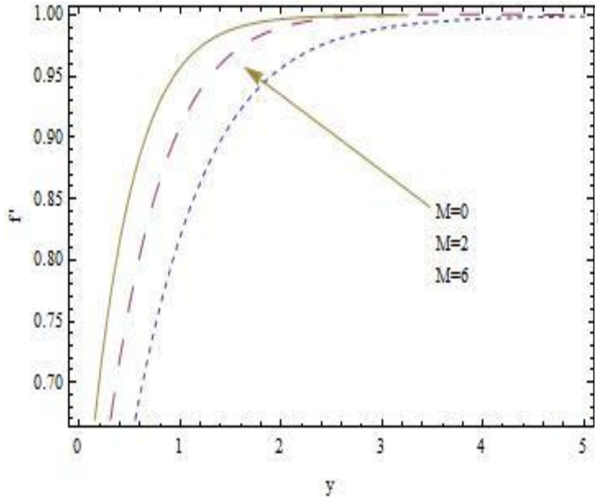

Fig. 8 Dimensionless velocity f' **for different values of** M **when** $n=0, \alpha=1, \Pr=1, R=5, \gamma=0.5, K=0.5$

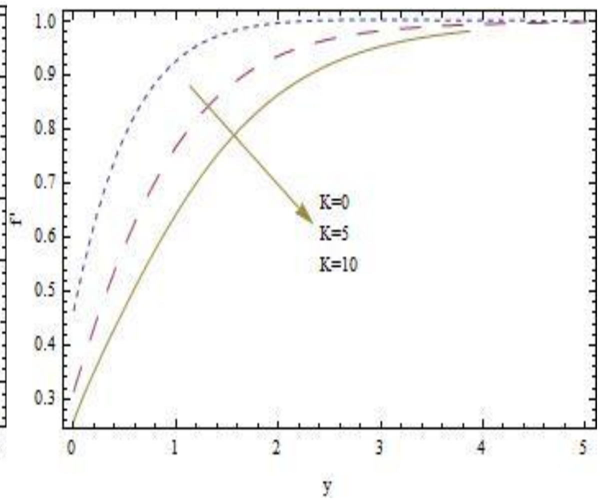

Fig. 9 Dimensionless velocity f' **for different values of** K **when** $M=1, n=0, \alpha=1, \Pr=1, R=5, \gamma=0.5$

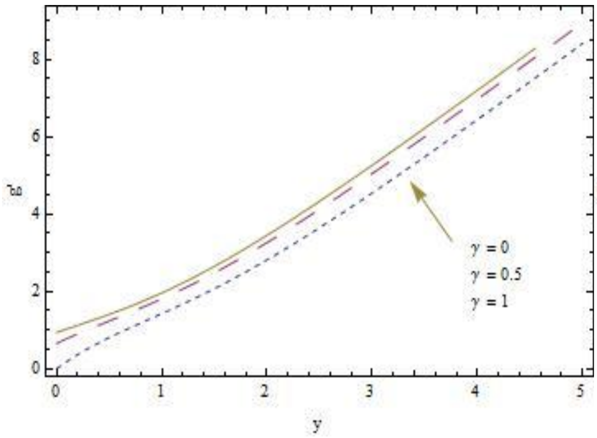

Fig. 10 Dimensionless velocity g' **for different values of** γ **when** $M=1, K=1, n=0, \alpha=1, \Pr=1, R=5$

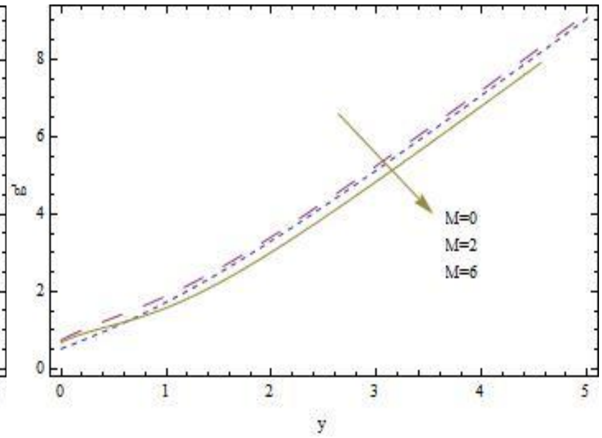

Fig. 11 Dimensionless velocity g' **for different values of** M **when** $n=0, \alpha=1, \Pr=1, R=5, \gamma=0.5, K=0.5$

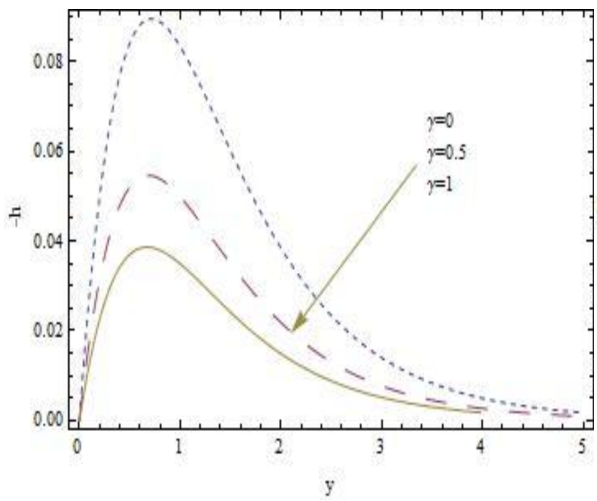

Fig. 12 Dimensionless angular velocity $-h$ **for different values of** γ **when** $M=1, K=1, n=0, \alpha=1, \Pr=1, R=5$

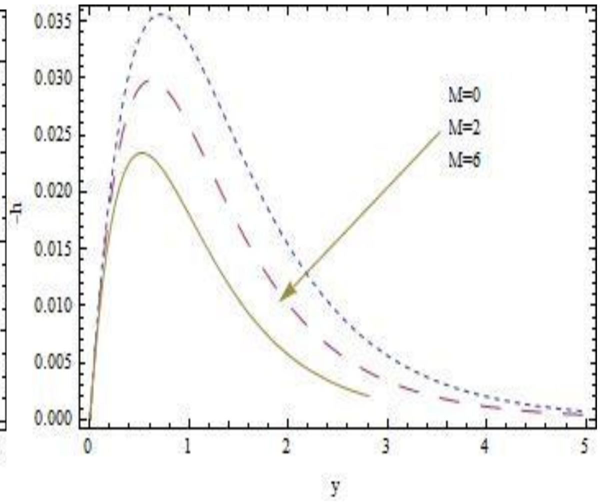

Fig. 13 Dimensionless angular velocity $-h$ **for different values of** M **when** $n=0, \alpha=1, \Pr=1, R=5, \gamma=0.5, K=0.5$

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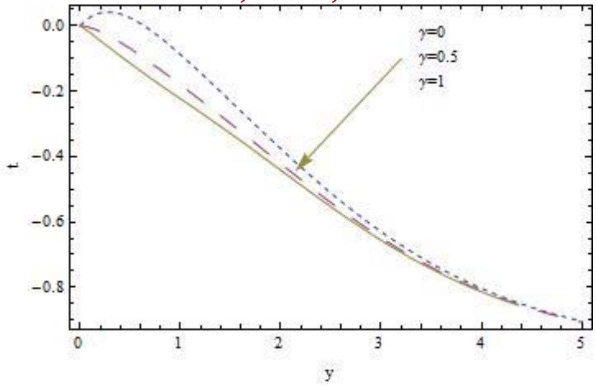

Fig. 14 Dimensionless angular velocity t for different values of γ when
$M=1, K=1, n=0, \alpha=1, \Pr=1, R=5$

VI. Conclusion

In this paper, we investigated the MHD oblique stagnation-point flow and heat transfer of a micro polar fluid towards to a moving plate with radiation under the velocity slip condition. Using one transformation, the partial differential equations are simplified into ordinary ones which are solved by HAM. The paper discussed the effects of γ, K, M, $\Pr$, R on velocity f', g'angular velocity $-h$, t and temperature θ. It can be found that f', g' increase as γ increases, while $-h$, t and θ decrease. Temperature θ increases as K increase while decreases with increasing $\Pr$ and R.

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REFERENCES

[1] M. Reza, A. S. Gupta, "Steady two-dimensional oblique stagnation-point flow towards a stretching surface," Fluid Dynamics Research, vol. 37 (5), pp.334-340, 2005.

[2] T. Mahapatra, S. Dholey, and A. S. Gupta, "Oblique stagnation-point flow of an incompressible visco-elastic fluid towards a stretching surface," International Journal of Non-linear Mechanics, vol. 42, pp. 484-499, 2007.

[3] Y. Y. Lok, I. Pop, and A. J. Chamkha, "Non-orthogonal stagnation-point flow of a micro polar fluid," International Journal of Engineering Science, vol. 45, pp. 173-184, 2007.

[4] T. Grosan, I. Pop, C. Revnic, and D. B. Ingham, "Magneto hydrodynamic oblique stagnation-point flow," Mechanical, vol. 44, pp. 565-572, 2009.

[5] D. Li, F. Labropulu, and I. Pop, "Oblique stagnation-point flow of a viscoelastic fluid with heat transfer," International Journal of Non-linear Mechanics, vol. 44, pp. 1024-1030, 2009.

[6] Y. Y. Lok, I. Pop, and D. B. Ingham, "Oblique stagnation slip flow of a micro polar fluid," Mechanical, vol. 45, pp. 187-198, 2010.

[7] I. Husain, F. Labropulu, and I. Pop, "Two-dimensional oblique stagnation point flow towards a stretching surface in a viscoelastic fluid," Central European Journal of Physics, vol. 9, no.1, pp. 176-182, 2011.

[8] T. R. Mahapatra, S. K. Nandy, and A. S. Gupta, "Oblique stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation," Mechanical, vol. 47, pp.1325-1335, 2012.

[9] M. A. A. Mahmoud, S. E. Waheed, "MHD stagnation point flow of a micro polar fluid towards a moving surface with radiation," Mechanical, vol. 47, pp. 1119-1130, 2012.

[10] S. J. Liao, "On the homotopy analysis method for nonlinear problems," Applied mathematics and Computation, vol. 147, pp. 499-513, 2004.

[11] S. J. Liao, "An optimal homotopy-analysis approach for strongly nonlinear differential equations," Communications in Nonlinear Science Numerical Simulation, vol.15, pp.2003-2016, 2010.

ISSN: 2319-5967
ISO 9001:2008 Certified
International Journal of Engineering Science and Innovative Technology (IJESIT)
Volume 2, Issue 2, March 2013

[12] S. J. Liao, A. Campo, "Analytic solutions of the temperature distribution in Blasius viscous flow problems," Journal of Fluid Mechanics, vol. 453, pp. 411-425, 2002.

[13] S. J. Liao, "A new branch of solutions of boundary-layer flows over an impermeable stretched plate," International Journal of Heat and Mass Transfer, vol. 48, pp. 2529-2539, 2005.

[14] S. J. Liao, "An analytic solution of unsteady boundary-layer flows caused by an impulsively stretching plate," Communication in Nonlinear Science and Numerical Simulation, vol. 11, pp.326-339, 2006.

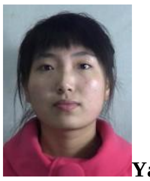
[15] S. J. Liao, "On the analytic solution of magneto hydrodynamic flows of non-Newtonian fluids over a stretching sheet," Journal of Fluid Mechanics, vol. 488, pp. 189-212, 2003.

[16] L. C. Zheng, J. J. Niu, X. X. Zhang, and Y. T. Gao, "MHD flow and heat transfer over a porous shrinking surface with velocity slip and temperature jump," Mathematical and Computer Modeling, vol. 56, pp. 133–144, 2012.

[17] L. C. Zheng, L. J. Wang, and X. X. Zhang, "Analytic solutions of unsteady flow and heat transfer on a permeable stretching sheet with non-uniform heat source/sink," Communications in Nonlinear Science and Numerical Simulation, vol. 16, pp. 731-740, 2011.

[18] J. Zhu, L. C. Zheng, and X. X. Zhang, "Homotophy analysis method for hydro magnetic plane and axisymmetric stagnation-point flow with velocity slip," World Academy of Science, Engineering and Technology, vol. 63, pp. 151-155, 2010.

AUTHOR BIOGRAPHY



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