Abstract—This paper describes a signal-based speed estimation of 3-Phase Brushless Direct Current (BLDC) motor using the relationship between commutation signal and Back Electromotive Force (Back-EMF). Both signals can directly be measured from each phase of motor’s terminal. The relationship function between both signals is approximated and filtered to find Half of Phase Voltage-Crossing Point. Genetic Algorithm (GA) is applied to estimate coefficients of the polynomial equation of relation between the variables $\sigma$, $\delta$, $\beta$ and commutation signal’s period. The results of estimated speed are accurate more than 97% and the average of overhead time is less than 9.0 ms.

Index Terms—3-Phase BLDC Motor, Speed Estimation, Genetic Algorithm.

I. INTRODUCTION

The Brushless Direct Current (BLDC) motor has been used increasingly in both industrial and household applications [1]-[12] such as CD player, hard disk drives, medical tools, air conditioners etc. This is because the BLDC motor has better efficiency, good reliability, low maintenance, and power density. Presently, detection of speed and position of BLDC has two approaches. Firstly, the rotor’s commutation signal is obtained by using the Hall-effect sensor. Secondly, by using the zero-crossing detection circuit, the rotor commutation signal of each phase can be estimated [3]-[7]. Mostly, the zero-crossing detection circuit uses op-amp to compare phase Back-EMF voltage with half of supply voltage. Therefore, there are many software-based methods proposed to find the zero-crossing of Back-EMF voltage for reducing the addition circuit. An artificial neural network (ANN) and a model reference adaptive system are used for finding a zero-crossing of Back-EMF in which the performance of the estimation is based on the properties of the motor [12]. The relation between the size of back-EMF voltage and the motor speed are applied to control the speed of motor using fuzzy logic in high speed range (more than 10,000 rpm) [13]. The artificial neural network is deployed to estimate the Back-EMF from voltage and current of each phase in low speed range [9]. Power rate reaching law sliding mode technique is applied to speed control for the inner-loop current control of BLDC motor and rejected load disturbance [14]. This paper presents the relationship of Back-EMF voltage, which is included within each phase voltage of motor. Phase voltage can be measured directly from each terminal of the motor. The function to estimate commutation signal from phase voltage is then created by using artificial intelligence techniques to find relations between each speed and commutation signal’s function. The resulting commutation signal’s frequency is then used as a frequency cut-off of low-pass filter for estimating Back-EMF signal from phase voltage. The filtered phase voltage signal or Back-EMF estimation can increase accuracy for estimating speed. The testing speed ranges in this work are approximately from 2,000 to 7,000 rpm. In this study we demonstrated a mathematical modeling of the BLDC motor and its electrical drives are described in Section II. Section III presents speed estimation procedure for the BLDC motor. Section IV and V are experimental results and conclusions.

II. PRINCIPLE OF BLDC MOTOR

A three-phase brushless DC motor has three induction coils with equivalent circuit of BLDC motors as described in the following section.

A. Mathematical Model of the BLDC Motor

The equivalent circuit of BLDC motor is shown in Fig.1. The dynamic equations of BLDC motor may be expressed in matrix form:
The electromagnetic torque developed by the motor is given by
\[ T_e = E_a I_a + E_b I_b + E_c I_c \]  
(2)
The moment of inertia includes all inertia connected to the motor shaft, and damping constant includes the air friction and the bearing friction. Therefore, the torque equation is
\[ J \frac{d\omega}{dt} + B\omega = T_e - T_l \]  
(3)
Where \( V_a, V_b, V_c \) are phase voltage [V], \( I_a, I_b, I_c \) are phase current [A], \( E_a, E_b, E_c \) are phase back EMF [V], \( R \) is phase resistance [Ω], \( L \) is self-inductance [H], \( M \) is mutual inductance [H], \( T_e \) is electromagnetic torque [N.m], \( T_l \) is load torque [N.m], \( \omega \) is angular velocity [rad/s], \( B \) is damping constant, [N.s/m], \( J \) is moment of inertia [kg.m²].

**B. The BLDC Motor Control**
BLDC motors are driven by a three-phase inverter with six-step commutation sequence. The inverter is commutated every 60 degree electrical when Ha, Hb and Hc are Hall effect sensor signal of each phase as shown in Fig.2.
C. BLDC Motor’s Back EMF

The 2 coils are induced to rotate through a magnetic field; the induced voltage is created in this coils, call that “Back Electromotive force” or “Back-EMF”. The relation between Back-EMF signal and Hall effect sensor of each phase is shown in Fig. 3.

III. SPEED ESTIMATION PROCEDURE

The phase angle of half point of Back-EMF voltage of each phase lags from Hall effect sensors signal is 30 degrees. The half point of this Back-EMF voltage can be estimated by half of phase voltage of motor [4] as shown in Fig. 4. The speed estimation procedure for the BLDC motor is described in next head respectively and speed estimation scheme is shown in Fig. 5.

A. Define Band and Select Signal Range

The half point of Back-EMF crossing includes the half point of phase signal voltage \( V_n \) in which a band of interesting signal must be defined as in equation (4) before estimating the speed.

\[
V^{(1)}_i = \begin{cases} 
\frac{V_n}{2}, & \frac{V_n}{2} - \frac{\delta}{2} \leq V_i \leq \frac{V_n}{2} + \frac{\delta}{2} \\
0, & \text{otherwise}
\end{cases}
\]  

(4)

Where \( \delta \) is window size of interesting band, \( V_i \) is phase voltage at \( I \) and \( V_n \) is supply voltage.

The selected signal is demonstrated in Fig. 6. The signals that pass the selected band depend on the window size of interesting band \( (\delta) \).

B. Find the Selected Signal’s Edge

Once the selected signal is defined, the envelope of the selected signal can be found by Moving Average (MA) as shown in equation (5). The envelope of the selected signal in Fig. 7 shows the highest density area of the half point of phase voltage crossing.

\[
V^{(2)}_i = \frac{\sum_{j=i-\sigma}^{i-1} V^{(1)}_j}{\sigma}
\]  

(5)

Where \( V^{(1)}_j \) is selected signal at \( j \), \( V^{(2)}_i \) is MA filtered signal at \( i \), and \( \sigma \) is window size of MA.
C. Find the Selected Signal’s Edge

The selected signal which is filtered by the MA function shows the highest density area of the half point of phase voltage crossing. The local maximum in this area can be estimated by (6) and (7). The result is shown in Fig. 8.

\[
L_{\text{max},i} = \begin{cases} 
V_{i}^{(2)} & , V_{i}^{(2)} > L_{\text{max},i-1} \\
L_{\text{max},i} & , \text{otherwise}
\end{cases}
\]  

(6)

\[
s_{i-\beta} = \begin{cases} 
1 & , L_{\text{max}} \in [V_{i-\beta}^{(2)}, V_{i}^{(2)}] \\
0 & , \text{otherwise}
\end{cases}
\]  

(7)

where
- \(L_{\text{max},i}\) is local maximum
- \(s_{i-\beta}\) is estimated commutation signal at \(i-\beta\)
- \(V_{i}^{(2)}\) is filtered signal at \(i\)
- \(\beta\) is previous time range from \(i\)
D. Estimate Back-EMF by Low-Pass Filter

The phase voltage signal is included in the Back-EMS signal. Both signals can be separated by using low-pass filter with frequency cut-off derived from the above method. This work uses Butterworth low-pass filter to separate Back-EMF from phase voltage signal as shown in Fig. 9.

E. Speed Calculation

The rotor speed can be calculated from frequency of estimated commutation of each phase using equation (8).

$$n = \frac{f \times 120}{Z_{pol}}$$  \hspace{1cm} (8)

where $n$ is motor shaft speed [rpm],

$f$ is frequency of commutation signal [Hz],

$Z_{pol}$ is number of motor’s magnetic poles

<table>
<thead>
<tr>
<th>Population</th>
<th>$e_r, %$</th>
<th>$e_\delta, %$</th>
<th>$e_\beta, %$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>198.4</td>
<td>63.60</td>
<td>133.97</td>
</tr>
<tr>
<td>100</td>
<td>3.37</td>
<td>75.36</td>
<td>18.03</td>
</tr>
</tbody>
</table>

Table I Error of the Fitness Curve from GA
In each commutation frequency or each speed, variables \( \sigma \), \( \delta \), \( \beta \) in each step are different. Therefore, the relation between variables \( \sigma \), \( \delta \), \( \beta \) for each speed must be calculated to be able to use in all speed range. The Genetic Algorithm (GA) [15] is applied to estimate coefficients of the polynomial equation of relation between the variables \( \sigma \), \( \delta \), \( \beta \) and commutation signal’s period.

The root mean square error of each parameter \( e_\sigma \), \( e_\delta \), \( e_\beta \) decreases when population size of each generation increases as shown in Table I and Fig. 10. The fitness curve from GA is more accurate of relation between \( \sigma \), \( \delta \), \( \beta \) and commutation signal’s period. The calculation time, however, increases also.

The relationship between \( \sigma \), \( \delta \), \( \beta \) and cutoff frequency for each speed is shown in Table II. Relation between \( \sigma \), \( \delta \), \( \beta \) and the previous commutation signal’s period \( T_{i-1} \) is shown in equation (9)

\[
\begin{bmatrix}
\sigma \\
\delta \\
\beta
\end{bmatrix} =
\begin{bmatrix}
0 & -0.0192 & 5.0974 \\
0.0002 & -0.0894 & 35.2124 \\
0.0004 & 0.0802 & 58.6711
\end{bmatrix}
\begin{bmatrix}
T_{i-1}^2 \\
T_{i-1} \\
1
\end{bmatrix}
\]  

(9)
In experiment, the BLDC motor is MAXON EC 45 Flat motor 45 mm, brushless, 50 Watt with the data as shown in Table III. The motor control is MAXON 1-Q-EC Amplifier AECS 35/3. The experimental setup is used Manchester M7 9RH for speed measurement, Tektronix TDS 1001B 40 MHz for data display and signal conditioning by MATLAB. The speed estimation method is divided into two steps. The first step is to find the frequency of the commutation signal using the each function in section III. The resulting frequency can be used to estimate speed but the result has more error. Therefore, the commutation signal’s frequency is known, the frequency cut-off of Butterworth low-pass filter can be applied to get the high accuracy for speed estimation.

Table III  Motor Data

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_v$</td>
<td>Nominal voltage</td>
<td>24.0 V</td>
</tr>
<tr>
<td>$n$</td>
<td>No load speed</td>
<td>6800 rpm</td>
</tr>
<tr>
<td>$I_{max}$</td>
<td>Nominal current (max. continuous current)</td>
<td>2.58 A</td>
</tr>
<tr>
<td>$R$</td>
<td>Terminal resistance phase to phase</td>
<td>1.03 Ω</td>
</tr>
<tr>
<td>$L$</td>
<td>Terminal inductance phase to phase</td>
<td>0.450 mH</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Torque constant</td>
<td>33.5 mNm / A</td>
</tr>
<tr>
<td>$J$</td>
<td>Rotor inertia</td>
<td>135 gcm$^2$</td>
</tr>
<tr>
<td>$P$</td>
<td>Pole permanent magnet</td>
<td>16</td>
</tr>
</tbody>
</table>

Fig.11 Output signal at speed 3,785 rpm  
Fig.12 %error of estimated speed before filtering at speed 3,785 rpm  
Fig.13 Phase voltage and filtered signal at speed 3,785 rpm  
Fig.14 %error of estimated speed at speed 3,785 rpm
A. Low Speed Estimation

Low speed range, Hall effect sensor signal, phase voltage signals and estimated signals at speed 3,785 rpm in Fig. 11, show that the signal is estimated to have periods near the period of Hall effect sensors signals. The parameters σ, δ and β at speed 3,785 rpm are 10, 30 and 15, respectively. The percentage error of estimated speed without using the low-pass filter shows in Fig 12. Once the frequency from previous step is used as the cutoff frequency of low pass filter, the frequency of the back-EMF can be estimated as shown in Fig. 13. The frequency of the back-EMF is used to estimate for speed again which is more accurate but overhead time of calculation increases. The percentage error of the estimated speed is shown in Fig. 14.

B. High Speed Estimation

High speed range, Hall effect sensor signal, phase voltage signals and estimation signals at speed 6,885 rpm in Fig. 15, show that the signal is estimated to have periods nearby the period of Hall effect sensors signals. The parameters σ, δ and β at speed 6,885 rpm are 8, 25 and 10, respectively. The percentage error of estimation speed without using the low-pass filter is shown in Fig 16. Once the frequency from previous step is used as the frequency cut-off of low pass filter, the frequency of the Back-EMF can be estimated as shown in Fig. 17. The approximate of frequency of the Back-EMF is used to estimate for speed again which is more accurate but overhead time of calculation increases. The percentage error of estimated speed is shown in Fig. 18. Fig.19 and Table IV shows the estimated speed in range of 2,000 to 7,000 rpm. At high range overhead time are decreased and the estimated speed error is less than the lower speed range. The damping force has effected in the low speed range more than high speed range. Thus, the relationship between σ, δ, β and commutation signal’s period for low speed must be improved.
This paper demonstrates speed estimation of 3-phase BLDC motor using Genetic Algorithm. The relationship between commutation signal and Back-EMF can be estimated speed of BLDC motor. The performance of the proposed method is verified by the experimental results in a speed range of 2,000 to 7,000 rpm. The proposed method can reduce the cost of speed measuring and can use in speed control system. The overhead time is less than 9.0 ms and the average percentage error of estimated speed is less than 2.

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REFERENCES


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